The Anatomy of Financial Exposures *

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Abstract

Financial stability depends on the network of financial exposures. Even though its anatomy is usually taken as exogenous, it reacts not only to changes in the environment but also to changes in regulations introduced to tame financial fragility. We construct a model with heterogenous banks that choose how much to insure through derivative contracts and whether to be exposed to a centralized (CCP) or bilateral (OTC) counterpart, weighing the collateral and transparency costs of these choices. By modeling the optimal decisions of atomistic members of finite banks, we use a *network taking* approach that bypasses strategic considerations and allows us to simultaneously capture optimal contracts and the endogenous network of financial exposures among banks. We characterize how the equilibrium network of financial exposures changes with capital requirement regulations, and how regulations may backfire if not taking this endogenous reaction into account.

Keywords: Central Counterparty (CCP), Over-the-counter Trading (OTC), Interbank Networks, Information Transparency, Network Reactions

JEL Classifications: G20, E50, N22

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1 Introduction

The recent global financial crisis had its epicenter at the malfunctioning of derivative markets, most notably credit default swaps (CDS) that were insufficient to cover the wave of potential defaults channeled through a large and complex network of derivative exposures among financial counterparties. This experience motivated the implementation of a flurry of regulations oriented towards putting constraints on the buildup of financial systemic risk and the likelihood of financial crises.

Two of the most salient objectives of these new regulations involved (i) a greater oversight over derivatives transactions, particularly those executed via over-the-counter (OTC) markets (this is, bilateral transactions of non-standard derivatives) and (ii) a greater level of transparency about these transactions. A controversial tool created to tackle both objectives was increasing the risk weights applicable towards computing capital requirements of derivatives transacted in relatively less transparent OTC markets, whereas reducing the risk weights applicable to those cleared through a relatively more transparent central counterparty clearing house (CCP).[1]

While these regulations have the explicit motivation of changing the structure of financial exposures, reducing their complexity, and inducing less contagion, we lack a tractable way to endogenize the network structure in response to environmental and regulation changes. In this paper, we construct an endogenous network model of interbank derivative exposures in which banks value the opacity offered by OTC contracts, but do not internalize their systemic danger via coordination failures.

In our model, banks house bankers who sign derivative contracts on behalf of their banks. Random pairs of bankers meet, and there are gains for them to sign derivative contracts for insurance purposes, for instance, because of asymmetric risk positions. If the derivative contract is observable, the bank "buying" insurance reveals that it holds a riskier asset than the bank "selling" insurance. This revelation may have implications for the bank's funding possibilities if outside investors are less inclined to fund riskier banks (or they are willing to fund it by charging a corresponding higher rate). With a transparent contract, the bank in need of insurance faces a *trade-off insurance-funding* that discourages efficient use of insurance in the market. This captures an efficiency gain of opacity: it implements insurance without fear of affecting the allocation of funds in the economy. As long as a derivative traded in OTC markets is more opaque than a derivative traded via CCPs, as argued by regulators, there is an efficiency loss in using CCPs. This view rationalizes the scarce use of CCPs before recent regulatory changes.

Facing these transparency costs of clearing and the potential failure to secure financing, bankers decide whether to sign insurance contracts , and if they do, they decide whether to clear the contract through a CCP. The aggregation of positions in contracts signed by bankers form

[1] See BCBS and IOSCO (2015) "Margin requirements for non-centrally-cleared derivatives" Technical report, BIS and OICU-IOSCO, Basel, Switzerland, for a discussion of this regulatory change.

the exposures between banks.

Our paper provides a novel and tractable way to capture changes in the network of financial exposures in response to changes in the environment and regulations. Instead of studying the strategic considerations of large agents (banks) to form their network of connections, we study the incentives of their atomistic members, who individually do not internalize their role in shaping the network but collectively end up determining its structure.

These exposures can lead to contagion across banks that is self-fulfilling, which we call a *coordination failure*. To fix ideas about the possibility of coordination failures in the absence of CCPs, take three banks A, B and C, that have developed a network of exposures. Assume that A is a net insurance seller to B, B to C and C to A, all for the same net amounts. If A can not pay B, then B can not pay C, then C can not pay A, then A can not pay B. Coordination failures trigger a self-fulfilling collapse on derivative networks. In the presence of central clearing, however, the implicit multilateral netting through bilateral netting with the CCP eliminates all exposures, and then the possibility of coordination failures disappears – a benefit of central clearing that is not internalized by banks and that rationalizes new regulations.

Despite its potential to completely eliminate coordination failures if adopted, widespread clearing is not mandated but instead induced by applying less risk weights in capital regulation to contracts that are cleared. In a world with heterogenous banks, however, how this asymmetric regulation affects the whole network of financial exposures and, ultimately, its stability is unclear. Understanding the endogenous reaction of optimal derivative contracts and the network of financial exposures is the goal of this paper.

We show that large banks operate large volumes of collateral, write many derivative contracts and tend to position at the core of operations. These banks tend to be less constrained by regulation and value opacity relative more. We show that insurance buyers in the core do not clear their exposures, whereas insurance buyers in the periphery do. This asymmetric response reduces bilateral netting between the core and the periphery. In turn, the CCP becomes post-regulation heavily exposed to the core. Additionally, exposures within the core are unaffected as the core can raise sufficient capital to remain unconstrained. Consequently, the cycles in the core, which are the triggers of coordination failures, persist despite being the most important target for regulation. Additionally, the exposures to the core increase, primarily by transforming the exposures of the periphery into exposures of the CCP on the extensive margin and increasing these exposures on the intensive margin due to relaxed regulation of cleared exposures. This endogenous reshaping of the network of financial exposures, *both in the extensive and intensive margins* may render regulations counterproductive and, at best, irrelevant to curbing financial fragility.

Related Literature:

We consider a network formed by infinitesimal agents (bankers) who act on behalf of finite institutions (banks) and whose individual actions do not have an aggregate impact individually but do in the aggregate. This approach simplifies the analysis of contagion and, so, the analysis of network formation. The most notable examples using this method for tractable analysis of strategic network formation are Erol (2024) and Erol and García-Jimeno (2022). The use of continuous networks at large in economics is relatively recent. We note the robustness of our results based on the convergence results established by Erol et al. (2023) and Parise and Ozdaglar (2023), who show that the outcome of games on growing networks limits the outcome of the corresponding game played on the limit graphon. We combine these tools to solve the network anatomy in the extensive margin (who writes derivative contracts with whom) and also the effects on the intensive margin (how large these contracts are).

The imposition of new regulations in the U.S. and Europe trying to reduce the influence of OTC derivative markets through incentivizing the use of CCPs has induced renewed literature on their effects.[2] This literature has focused on different relevant aspects. Duffie et al. (2015) discuss how CCPs affect collateral demand in the system (given the heterogeneity of margin requirements), which may have important distributional consequences across intermediaries. Cont and Kokholm (2014) highlight that CCPs may reduce counterparty risk, but at the cost of reducing netting across asset classes. Part of this literature is also concerned about the resolution protocols of CCPs in distress and their potential systemic consequences, such as Duffie (2015), Bignon and Vuillemey (2019), Kuong and Maurin (2024), and Capponi et al. (2019).

Our focus is different. We explore incentives and trade-offs between costs and benefits of clearing, how regulations affect these tradeoffs, and how the network structure and systemic risk is changes in response. Low CCP risk weights create a tradeoff between bilateral and multilateral netting between the core and the periphery. Transparency makes the core avoid clearing of core-core exposures. The closest to our insight regarding transparency is highlighted in Spatt (2017), who concludes that transparency should consider liquidity needs and should not increase trading costs. We additionally highlight that simultaneous "success" in moving the periphery to clearing creates adverse unintended consequences in terms of systemic risk by exposing the CCP to the core, even in the hypothetical absence of a need to manage and govern CCPs' risks. Duffie and Zhu (2011) explores a substitution between bilateral netting and multilateral netting but they do not study incentives, the network structure, or systemic risk.

The literature on the role of transparency of CCPs, the lack thereof in OTC markets, is, however, scarcer. Babus and Kondor (2018) discuss how information flows through the network in OTC markets and how it affects trading, while Glode and Opp (2023) show that OTC markets

[2] A recent legal literature, such as McBride (2010) and Allen (2012), have also studied the effects of CCPs and their regulation.

can be rationalized in spite of larger frictions than centralized markets when traders' expertise is endogenous. In contrast to this literature, here we focus on the transparency of the contracts and its potential to reveal information about traders' types and assets' types. This is more in line with the positive view of opacity highlighted in Dang et al. (2017), who rationalize the banking opaque operations, and Gorton et al. (2025), who rationalize opaque and complex financial exposures as a way to increase funding capacities.

Our work is also connected to recent literature studying the functioning and the fragility of OTC markets, partly initiated by a search-theoretic approach applied to asset markets, such as Duffie et al. (2005) and Lagos and Rocheteau (2009). Afonso and Lagos (2015), study the functioning of federal fund markets by applying a search model to study how two banks get together and bargain bilaterally. Following this tradition, Atkeson et al. (2015) introduces entry and exit in OTC derivative markets and study the characteristics of the ensuing network. Even more recently, Hugonnier et al. (2020) study the role of heterogeneity and of search and bargaining frictions in these markets. In contrast with this work, instead of studying how banks in a network negotiate, we study the optimal contract of its members and what it implies for the aggregate observed network of financial exposures.

Finally, our paper is also a contribution to the recent literature on the unforeseen effects of government regulations and interventions to financial networks, such as Erol and Ordonez (2017) in terms of capital regulations and Anderson et al. (2019) in terms of public liquidity provision. In D'Erasmo et al. (2025) we show empirically, using confidential data, that in response to recent regulations inducing CCPs in the United States, there was an asymmetric response consistent with the implications of this model.

Section 2 describes the model. Section 3 characterizes the equilibrium network and the optimal insurance contracts in the absence of regulations. Section 4 introduces capital requirements and shows how the networks of financial exposures respond to changes in regulation. Section 5 introduces heterogeneity among banks, which displays a core-periphery structure and shows how capital requirements reshape the network and affect contagion. Section 6 concludes.

2 Model

2.1 Environment

In what follows, we maintain the following notation and exposition choices. All random variables are independent unless noted otherwise. Parametric assumptions made in the text are maintained from the point they are stated. Assumptions made inside results only apply to those results. When there is no risk of confusion, subscripts and superscripts are dropped to highlight the variables of interest and to reduce clutter.

Agency structure. There is a finite set of *banks*, each denoted by $u \in B$. Each bank's equity is owned by a representative *shareholder*, who can borrow from a representative *creditor* to finance a set of *projects*. Each bank houses a mass of *bankers*, each denoted by i, with b_i denoting i's bank. Each banker manages one of the bank's projects and additionally obtains, through the bank, access to an *investor* to finance an *investment* as a joint venture.

Investments are subject to an idiosyncratic shock, which introduces the need for insurance. Since investments are non-pledgeable, such insurance has to be backed by collateral. Projects are subject to an *aggregate shock* and so uninsurable, but are pledgeable, so can be used as *collateral* by bankers to insure against the shocks to investments. Insurance takes place through *(derivative) insurance contracts* between two bankers of different banks randomly matched. Bankers sign investment and insurance contracts on behalf of their banks. Bankers are compensated as a fraction of the profits they generate for their banks. These insurance contracts will generate an interbank network of liabilities. Details about projects and investments follow.

Projects. The creditor has deep pockets and *lends* to u using a debt contract at a market rate. Each project requires m' lending to initiate and a banker's management to mature. For a project managed by banker $i \in u$, the net rate of return to the bank is $(\alpha + \zeta_i)c_u - m$ where c_u is the project's baseline return, $\alpha \sim U[\underline{\alpha}, \overline{\alpha}]$ is an *aggregate shock* to projects in economy, $\zeta_i \sim U[-Z, Z]$ is an *idiosyncratic shock* to banker i, and m is the repayment promised to creditors. The fair value of projects will serve as *collateral* for bankers.

Investments. Each banker i has access to an investment opportunity that requires 1 unit of funds to finance. There is an infinite supply of investors that may choose to contact the banker to provide the funds. If an investor matches with banker i, they are locked in, and we denote the investor as n_i. The investor can, however, walk away from the investment before it matures based on an outside option that pays a random return w_i, and generates utility $\omega_i := V_I(w_i) \sim U[0, \overline{\omega}]$, for some $\overline{\omega} > 0$.[3] When indifferent, n_i uses this outside option. The banker and the investor observe the outside option after they match but before the project starts.

If financed, i's investment yields a random *return* $r_i \in \{r, 0\}$ (*success* or *failure*) depending on the *quality* $q_i \in \{0, 1\}$ of the managing banker i (*bad* or *good*). The banker i is good with probability γ_i. While the banker knows his own quality, the investor does not. The banker i investment's *success probability* is $\sigma_i = \sigma_{i0} + q_i(\sigma_{i1} - \sigma_{i0})$, with $\sigma_{i1} > \sigma_{i0} > 0$ constants. That is, good bankers are better at investing than bad bankers. We assume the following split of returns in case of success. A *share* $s \in (0, 1)$ goes to n_i, a share $s' < 1 - s$ goes to banker i, and the remainder $1 - s - s'$ goes to the bank. We assume $\omega^* := V_I(sr) < \overline{\omega}$, so there are investors with outside options high enough not to finance even projects that are certain to succeed.

Insurance needs. Each banker has utility function $V_B(x) = x + \theta \min\{\beta, x\}$ where $\theta, \beta > 0$ are constants. This utility function displays global risk aversion around the "*breaking point*" β

[3]The corresponding distribution of w_i has CDF $F_w(w_i) = V_I(w_i)\overline{\omega}^{-1}$ for all $w_i \in [0, V_I^{-1}(\overline{\omega})]$.

and local risk neutrality.[4] Risk aversion creates incentives for diversification, up to an extent determined by the *steepness* θ, which can also be seen as a measure of risk aversion. Under this utility function, there are potential insurance gains if a banker has less than β return and another banker has more than β. We assume $rs' > 2\beta$ so that returns to two bankers from one successful investment is sufficient to give both β with a suitable insurance contract.

Insurance contracts. Some pairs of bankers are matched bilaterally for an opportunity to insure each other. The matching protocol is described in detail in Appendix 3.2.3. The insurance contracts are signed on behalf of the banks and secured by their collateral. A (*derivative*) *insurance contract* (d_{ij}, d_{ji}) between two bankers i and j is contingent on the outcomes of investments of i and j. The value $d_{ij}(\tilde{r}_i, \tilde{r}_j)$ describes the "payment from j to i", this is the liability of b_j (banker j's bank) and the asset of b_i (banker i's bank) as a function of investment returns $(\tilde{r}_i, \tilde{r}_j)$. A fraction s'' of the final return from the insurance contracts is accounted towards the bankers who signed the contract. The remaining share $1-s''$, is retained by the bank. If the pair does not have a contract, we denote this with $(d_{ij}, d_{ji}) \equiv \mathbf{0}$. As bankers aim to achieve β but their share from insurance is s'', a banker with an investment failure would require a payment $\kappa \equiv \beta/s''$, so to guarantee consuming β.

Each banker in a matched pair decides whether to insure or not. If both bankers decide to insure, they sign an *optimal contract* which is defined as a feasible contract that maximizes the sum of the pair's expected utilities subject to individual rationality.

Collateral for Insurance. Liabilities in each contract must be fully collateralized. The fair value of unposted collateral earns the banker $\xi\theta s''$ utils of private gain per dollar of unposted collateral. We call ξ (normalized opportunity) *cost of collateral*.[5]

Insurance Platforms Insurance contracts can be channeled through two possible platforms: $P \in \{O, C\}$. O represents OTC platforms, which are characterized by bilateral contracts (d_{ij}, d_{ji}). C represents CCP platforms, which are characterized by the intermediation of a central clearing counterparty, which *novate* the contract, meaning that the contract (d_{ij}, d_{ji}) is replaced with two identical contracts, $(d_{iC}, d_{Ci}) \equiv (d_{ij}, d_{ji})$ between b_i and the CCP, and $(d_{Cj}, d_{jC}) \equiv (d_{ij}, d_{ji})$ between the CCP and b_j. These novated contracts are still contingent on the success of investments of i and j, but the CCP is not a bank and does not have investments. The CCP simply inserts itself as the counterparty to both of the original counterparties.

When a large number of contracts are novated, the CCP can "clear" complex exposures that would arise across banks simply in its own balance sheet. Strictly speaking, then, all contracts are OTC when signed. If a contract is not novated after signing, we say the contract is *kept OTC*, and we use *OTC exposures* for the corresponding exposures between banks.

Investor information. As information about contracts may "leak" to the market, counter-

[4] The advantages of this specification to study insurance are discussed by Dang et al. (2017).

[5] The private gains are utils to simplify the contracts. This can be seen as management returns materializing at a different date.

parties can revise their insurance decision to conceal information before it leaks. Formally, after two matched bankers i and j sign a contract, nature determines *signals* $(\iota_{ij}^C, \iota_{ij}^O) \in \{0,1\}^2$ and $(\iota_{ji}^C, \iota_{ji}^O) \in \{0,1\}^2$, capturing the information that can potentially be inferred by investors about bankers' qualities. The signal ι_{ij}^C is about i's quality if the contract is novated through the CCP and ι_{ij}^O is about i's quality if the the contract is kept OTC.

The pair i and j both observe $(\iota_{ij}^C, \iota_{ij}^O)$ and $(\iota_{ji}^C, \iota_{ji}^O)$ before settlement. After observing the signals , the pair (this is both bankers in the joint decision) either novate the contract through the CCP, keeps the contract OTC, or annuls the contract.[6] If the contract is novated, n_i observes $\iota_{ij} = \iota_{ij}^C$. If the contract is kept OTC, n_i observes $\iota_{ij} = \iota_{ij}^O$. If the contract is annulled n_i observes $\iota_{ij} = 1$, this is no signal. We call (the observed signal) ι_{ij} the *investor information* regarding i. The investor can liquidate the investment without cost after observing the signals and updating his belief about the banker's type and hence having a better idea of the investment succeeds.

The signal about i's quality $(\iota_{ij}^C, \iota_{ij}^O)$ is (q_i, q_i) with probability τ_{ij}^O, and $(q_i, 1)$ with probability $\tau_{ij}^C > \tau_{ij}^O$. In words, with probability τ_{ij}^O an insurance contract reveals banker i's type regardless of whether it is novated or not, with probability τ_{ij}^C it reveals the type only if novated (in this case the signal is always 1 if OTC, and hence uninformative). With complementary probabilities, the signal does not reveal agent i's quality regardless of the contract platform. This structure captures the idea that monitoring and transparency attached to novation through a CCP results in superior information discovery for the market. This signal structure is convenient as, for a given platform $P \in \{C, O\}$, τ^P reflects the level of *transparency* implied by the use of the platform.

Payouts absent defaults. For investors, absent defaults, n_i gets paid $r_i s$ or w_i (achieving utility ω^* or ω_i), depending the choice between financing or not (the outside option).

For bankers, if the pair i and j has insurance, i gets paid $p_i = s' r_i + s''(d_{ij}(r_i, r_j) - d_{ji}(r_j, r_i))$. If i does not have insurance, he gets paid $p_i = s' r_i$. If n_i liquidates early, $p_i = 0$. Banker i's utility net of management gains is $V_B(p_i)$ less the cost of collateral $\xi \theta s''(r + \max_{(\tilde{r}_i, \tilde{r}_j)}\{d_{ji}(\tilde{r}_j, \tilde{r}_i)\})$.

For creditors, they get paid $m - m'$ per unit of project funded. For shareholders, the shareholder of bank u retains earnings $(\alpha + \zeta_i)c_u - m + r_i(1 - s) - p_i$ integrated over all bankers. The creditor and the shareholder are mechanical parts of the model, so we specify their utility functions only when we study welfare.

Consolidation and netting. Each bank consolidates its on- and off-balance sheet positions: projects, investments, liabilities to creditors and investors, and derivative insurance contracts.

Bank u's projects have asset value $\int_u (\alpha + \zeta_i) c_u \mathtt{d}i = \alpha \mu_u c_u$.[7] We denote $A_u = \mu_u c_u$. Bank u's projects create liability $L'_u = m\mu_u$ to the creditor. The total returns from investments is $R_u = \int_{u_R} r_i \mathtt{d}i$ where u_R is the set bankers in u investing. The total promised share of return to

[6] The only role of assuming that bankers observe signals before the market is to avoid early liquidations on the path of play.

[7] Note that ζ_i is not correlated with i's financing.

the investors is $L''_u = sR_u$. These are the *senior liabilities*, adding up to $L_u = L'_u + L''_u$. With some abuse of notation, we call $\alpha A_u + R_u$ *senior assets*.

Similar integration identifies interbank assets and liabilities. Denote by i^* the matched banker for banker i in bank u. Also denote $i \in u_{Ov}$ if $i \in u$, $i^* \in v$, and the pair implemented a contract on OTC. Then $b_{i^*}//b_i$ must fulfill the payment of $d_{ii^*}(r_i, r_{i^*})//d_{i^*i}(r_{i^*}, r_i)$ directly to $b_i//b_{i^*}$. Denote $i \in u_{Cv}$ if the pair implemented a contract on CCP. Then $b_{i^*}//b_i$ must fulfill the payment of $d_{ii^*}(r_i, r_{i^*})//d_{i^*i}(r_{i^*}, r_i)$ to the CCP, and the CCP must fulfill the payment of the same amount to $b_i//b_{i^*}$. The total expected payment promise to b_i and by b_i are

$$D_{ii^*} = \mathbb{E}_{(\tilde{r}_i, \tilde{r}_i^*)}[d_{ii^*}(\tilde{r}_i, \tilde{r}_{i^*})], \quad D_{i^*i} = \mathbb{E}_{(\tilde{r}_{i^*}, \tilde{r}_i)}[d_{i^*i}(\tilde{r}_{i^*}, \tilde{r}_i)]$$

The gross interbank assets arising out of contracts between their bankers are called *exposures*. The gross *OTC exposure* of u to v is E_{uv} and the gross *cleared exposure* of u to v is E_{uCv}, given by[8]

$$E_{uv} = \int_{u_{Ov}} D_{ii^*} \mathtt{d}i, \quad E_{uCv} = \int_{u_{Cv}} D_{ii^*} \mathtt{d}i$$

The gross exposure of u to the CCP is then $E_{uC} = \sum_v E_{uCv}$. The gross exposure of the CCP to u is $E_{Cu} = \sum_v E_{vCu}$.

Institutions can have two sided exposures with each other. Opposing two sided exposures between two given counterparties are eliminated via *bilateral netting* (a master netting agreement between the institutions). Bilateral netting results in *net exposures*: the net exposure of institution f (a bank or the CCP) to f' is

$$E'_{ff'} \coloneqq \langle E_{ff'} - E_{f'f} \rangle$$

where the notation $\langle \cdot \rangle$ is the positive of a number:

$$\langle \cdot \rangle = \max\{0, \cdot\}$$

Note that netting does not affect equity as $x_1 - x_2 = \langle x_1 - x_2 \rangle - \langle x_2 - x_1 \rangle$ for any x_1, x_2. Also note that $\min\{E'_{ff'}, E'_{f'f}\} = 0$. This is, after netting, there are no two-sided exposures left.

The collection of these aggregate values with bilaterally netted exposures across all banks is called the *system S:*

$$S = (A_u, R_u, L_u, (E'_{uC}, E'_{Cu}), (E'_{uv})_{v \in B})_{u \in B}$$

[8]In fact, $E_{uv} = \int_{u_{Ov}} d_{ii^*}^{r_i r_i^*} \mathtt{d}i$. Under some regularity conditions this is equal to $\int_{u_{Ov}} D_{ii^*} \mathtt{d}i$. We assume such regularity throughout the paper.

2.2 Timing.

Stage 1 - Funding and lending: Creditors lend to banks, banks initiate projects and give it to a banker to manage. Investors match with bankers. Investors' outside options and bankers' qualities are drawn.

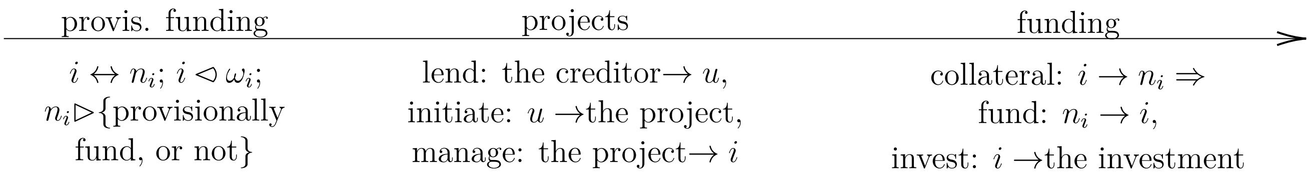

Figure 1: Funding and lending stage
"$\lhd$" indicates observing information. "$\rhd$" indicates choosing an action.
"$\to$" indicates mechanical components (with possibly trivial incentives at the background).

Stage 2 - Insurance and information: Bankers are matched. Matched bankers observe each others' quality. Then each pair of matched bankers decide to insure or not. If bankers in a pair both decide to insure, they sign an optimal contract. Then nature determines signals, which are observed by the corresponding pairs. Then each pair decides whether to novate their contract, keep it OTC, or annul it. Upon observing contracts and signals, each investor chooses to withdraw his funds or not. If an investor withdraws, the corresponding banker liquidates the investment, repays the investor, and the banker's insurance contract is annulled.

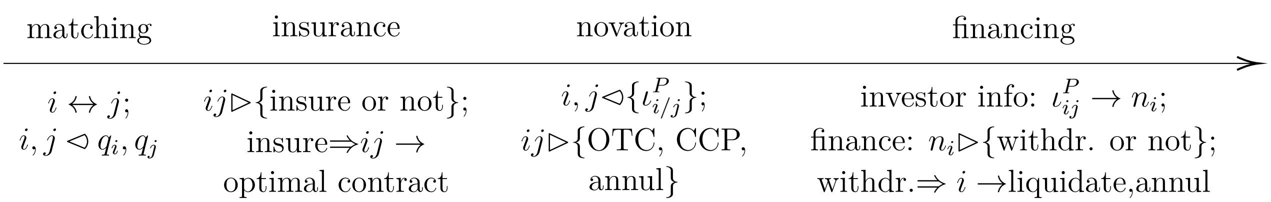

Figure 2: Insurance and information stage

Stage 3 - Consolidation and contagion: Banks consolidate their positions. Bilateral netting is executed. Aggregate shock is realized. Then, liquidations, maturity, and contagion (as we will describe next) materialize simultaneously. Payments to agents are made as per contractual promises, using seniority and proportional sharing rules as described.

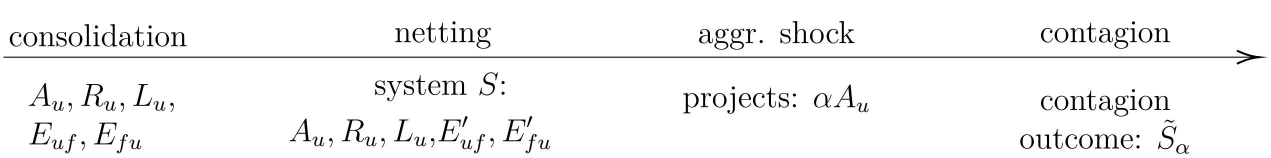

Figure 3: Consolidation and contagion stage

2.3 Contagion

We follow Eisenberg and Noe (2001) and Acemoglu et al. (2015) to model financial contagion. Intuitively, the aggregate shock α alters the value of all projects. Depending on the system, a bank may not be able to repay its promises (to creditors, investors and other banks) if it receives less return to projects than expected, given the promises obtained from other banks. This may force the *liquidation* of projects and investments (akin fire sales), reducing their value to fractions $\lambda_A < 1$ and $\lambda_R < 1$ respectively, and turning the bank insolvent.[9] If a bank defaults, seniority prioritizes liabilities to creditors and investors first, then interbank liabilities, then bankers (employees) and finally bankholders. Within each seniority, agents obtain payments proportional to the original net asset value.

More formally, let $E_f^{\rightarrow}$ be total *exposures to* institution $f \in B \cup \{C\}$ (this is, $f's$ interbank liabilities) and $E_f^{\leftarrow}$ total *exposures of* institution f (this is, f's interbank assets):

$$E_f^{\rightarrow} = \sum_{f' \in B \cup \{C\}} E'_{f'f}, \qquad E_f^{\leftarrow} = \sum_{f' \in B \cup \{C\}} E'_{ff'}$$

Notice $E_C^{\rightarrow} = E_C^{\leftarrow}$ since a novated contract induces the same exposure to and from the CCP. After the consolidation, netting, and the realization of the aggregate shock, absent any defaults, bank u's equity would be given by

$$Q_{u,\alpha} = \alpha A_u + R_u + E_u^{\leftarrow} - L_u - E_u^{\rightarrow}$$

When there are defaults, the recovered assets may not be enough to cover the returned liabilities. Formally, it is possible that

$$\underbrace{\alpha A_u + R_u + E_u^{\leftarrow}}_{\textit{Assets}} > \underbrace{L_u + E_u^{\rightarrow}}_{\textit{Liabilities}} > \underbrace{\lambda_A \alpha A_u + \lambda_R R_u + E_u^{\leftarrow}}_{\textit{Liquidated Assets}}$$

This is, u can default simply because it defaults. This is akin to a bank-run. We aim instead to study banks defaulting because of each other so we focus on self-fulfilling contagion rather than self-fulfilling bank-runs. For this, we assume away this possible multiplicity involving default, this is if no default is an equilibrium, we select that equilibrium.

Recovered assets and returned liabilities are determined simultaneously "during" contagion. Accordingly, we define *"intra-contagion" assets and liabilities* (we denote these variables with tildes) through endogenous *recovery and return rates* (rr). Recovered assets from senior assets after liquidation or maturity are $\tilde{A}_{u,\alpha} = \text{rr}^A_{u,\alpha} A_u$ and $\tilde{R}_{u,\alpha} = \text{rr}^R_{u,\alpha} R_u$. Recovered interbank assets of u are $\tilde{E}'_{uf} = \text{rr}^E_{f,\alpha} E'_{uf}$ and returned interbank liabilities of u are $\tilde{E}'_{fu} = \text{rr}^E_{u,\alpha} E'_{fu}$. The

[9] Creditors demand liquidation of investments and investors demand liquidation of projects simultaneously. Creditors and investors can not coordinate.

total recovered interbank assets are $\tilde{E}_u^{\leftarrow} = \sum_f \tilde{E}'_{uf}$ and total returned interbank liabilities are $\tilde{E}_u^{\rightarrow} = \sum_f \tilde{E}'_{fu}$. Returned senior liabilities to the creditor and investors are $\tilde{L}_{u,\alpha} = \mathrm{rr}^L_{u,\alpha} L_{u,\alpha}$. Then $\mathrm{RA}_{u,\alpha} = \alpha \tilde{A}_{u,\alpha} + \tilde{R}_{u,\alpha} + \tilde{E}^{\leftarrow}_{u,\alpha}$ and $\mathrm{RL}_{u,\alpha} = \tilde{L}_{u,\alpha} + \tilde{E}^{\rightarrow}_{u,\alpha}$.

When there are no defaults, all recovery and return rates are equal to 1.

Defaults are triggered by bad aggregate shocks and low interbank asset recovery. Formally, bank u is said to *default* if

$$Q^{\mathrm{def}}_{u,\alpha} \coloneqq \alpha A_u + R_u + \tilde{E}^{\leftarrow}_{u,\alpha} - L_u - E^{\rightarrow}_u < 0 \tag{1}$$

This condition uses the original values of liabilities and all assets other than interbank assets. It is assumed that liabilities are not renegotiated in response to contagion.

Since defaulting banks are forced to liquidate their projects and investments before maturity, recovery rates are

$$(\mathrm{rr}^A_{u,\alpha}, \mathrm{rr}^R_{u,\alpha}) = \begin{cases} (1,1) & \text{if } u \text{ does not default } (Q^{\mathrm{def}}_{u,\alpha} \geq 0) \\ (\lambda_A, \lambda_R) & \text{if } u \text{ defaults } (Q^{\mathrm{def}}_{u,\alpha} < 0) \end{cases}$$

As investors and creditors are paid first, up to the total assets:

$$\tilde{L}_{u,\alpha} = \min\{L_u, \mathrm{RA}_{u,\alpha}\}$$

Out of this aggregated amount $\tilde{L}_{u,\alpha}$, each investor and ex-ante creditor gets proportional payments: $\mathrm{rr}^L_{u,\alpha}$ fraction of the original liability. In particular, n_i is owed $r_i s$ but obtains just $\mathrm{rr}^L_{u,\alpha} r_i s$. Institutions are second in seniority, and pay each other proportional to how much each is owed up to their remaining funds:

$$\mathrm{rr}^L_{u,\alpha} = \frac{\tilde{L}_{u,\alpha}}{L_{u,\alpha}}, \qquad \mathrm{rr}^E_{u,\alpha} = \min\left\{1, \frac{\mathrm{RA}_{u,\alpha} - \tilde{L}_{u,\alpha}}{E^{\rightarrow}_u}\right\}, \qquad \mathrm{rr}^E_{C,\alpha} = \frac{\tilde{E}^{\leftarrow}_{C,\alpha}}{E^{\leftarrow}_{C,\alpha}}$$

After these more senior liabilities are fulfilled (perhaps partially), the bank has $\mathrm{RA}_{u,\alpha} - \mathrm{RL}_{u,\alpha}$, which are use to pay bankers $R_u s' + (E^{\leftarrow}_u - E^{\rightarrow}_u) s''$, up to those remaining funds with a proportional share. So the banker i gets paid $p_i \min\{1, \frac{\mathrm{RA}_{u,\alpha} - \mathrm{RL}_{u,\alpha}}{R_u s' + (E^{\leftarrow}_u - E^{\rightarrow}_u) s''}\}$. What remains is the bank equity.[10]

This completes the description of contagion. Given the system S and a shock α, for a solution to the system above, the resulting vector of corresponding components of the system is called a *contagion outcome*:

$$\tilde{S}_\alpha = (\tilde{A}_{u,\alpha}, \tilde{R}_{u,\alpha}, \tilde{L}_{u,\alpha}, (\tilde{E}'_{uC,\alpha}, \tilde{E}'_{Cu,\alpha}), (\tilde{E}'_{uv,\alpha})_{v \in B})_{u \in B}$$

[10]In principle, the CCP and the banks can have different seniorities. As it will become clear later, this is not important as only investors and ex-ante creditors receive positive payments from a bank when the bank defaults.

3 Characterization

3.1 Contagion Stage: Coordination failures

We now define coordination failures and characterize the importance of cycles for its existence.

Definition. A system with a realized aggregated shock is said to have a *coordination failure* if both *no bank* and *all banks* default are separate contagion outcomes of the system **and** pay 0 to each other in case of default. Formally, (S, α) has a coordination failure if there are two contagion outcomes $\tilde{S}_{\alpha_1}, \tilde{S}_{\alpha_2}$ such that $\tilde{Q}^{\text{def}}_{u,\alpha_1} < 0 \leq \tilde{Q}^{\text{def}}_{u,\alpha_2}$ **and** $\tilde{E}^{\rightarrow}_{u,\alpha,1} = 0$ for all banks u.

When there is a coordination failure, we select the outcome with default. Next, we characterize network conditions for the existence of a coordination failure.

Theorem 1. *Necessary conditions for the existence of a coordination failure are that the network i) has a directed cycle and ii) each bank is exposed to a cycle, either directly in the cycle or indirectly exposed to a bank in the cycle.*

Figure 4 portrays an example of the topological conditions for a coordination failure. All banks are on a subnetwork that is a cycle-rooted tree. Banks along directed cycles default in a jointly-self-fulfilling fashion (as in the earlier example with three banks). Then cascading defaults outward from such cycles.

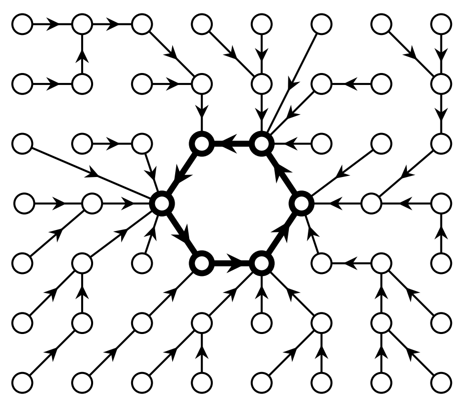

Figure 4: A cycle-rooted tree subgraph. Forest of cycle-rooted trees is a necessary condition for coordination failures.

Notice that the restriction of 0 payments in case of default is a strong restriction that we impose for tractability. Still, relaxing that constraint (what we can call a “weak coordination failure") still requires a cycle for existence.

It is easy to identify conditions for a coordination failure from the default condition (1).

$$E^{\rightarrow}_u > \alpha A_u + R_u - L_u \geq E^{\rightarrow}_u - E^{\leftarrow}_u \tag{2}$$

$$0 > \alpha \lambda_A A_u + \lambda_R R_u - L_u \tag{3}$$

The first inequality of condition (2) implies that a bank that receives no payments from other institutions defaults. The second that there is no default if no other bank defaults. The inequality of condition (3) guarantees that all banks defaulting and paying 0 to each other is indeed a contagion outcome.

Going forward, and to guarantee that all coordination failures involve 0 payments in case of default, we assume $\lambda_R < \max\{s\,,\, 1 - s'/s''\}$, which is sufficient when S arises endogenously. Intuitively, the maximal interbank liability $E_u^{\rightarrow}$ is formed for insurance against investments, and so it is endogenously smaller than a fraction of investment returns. This is, $E_u^{\rightarrow} < R_u \max\{1-s, s'/s''\}$.[11] Then, since interbank liabilities are junior liabilities, any defaulting bank in any contagion outcome makes 0 payment to other banks. Depending on whether there is a coordination failure or not, interbank recovery and return rates are either all 0 or all 1, and return rates for senior liabilities are $(\mathrm{rr}^L_{u,\alpha})_{u\in B} = \left(\frac{\alpha\lambda_A A_u + \lambda_R R_u}{L_u}\right)_{u\in B}$ or $(1)_{u\in B}$. We can use this to identify the probability coordination failures tractably.

Proposition 1. *There exists a (weak) coordination failure if and only if* $E_u^{\rightarrow} > \alpha A_u + R_u - L_u \geq E_u^{\rightarrow} - E_u^{\leftarrow}$. *Denoting the coordination failure cutoffs of a bank by*

$$\begin{aligned}\phi_u &:= \frac{E_u^{\rightarrow} + L_u - R_u}{A_u} \\ \phi'_u &:= \frac{E_u^{\rightarrow} - E_u^{\leftarrow} + L_u - R_u}{A_u}.\end{aligned} \tag{4}$$

There exists a coordination failure if and only if

$$\phi := \min_u \phi_u > \alpha \geq \max_v \phi'_v =: \phi' \tag{5}$$

Denoting $\Phi_u = F_\alpha(\phi_u) - F_\alpha(\phi')$, *the probability of a coordination failure is* $\Phi = \min_u \Phi_u$.

The coordination failure cutoffs ϕ and ϕ', and multiplicity of contagion outcomes are portrayed in Figure 5.

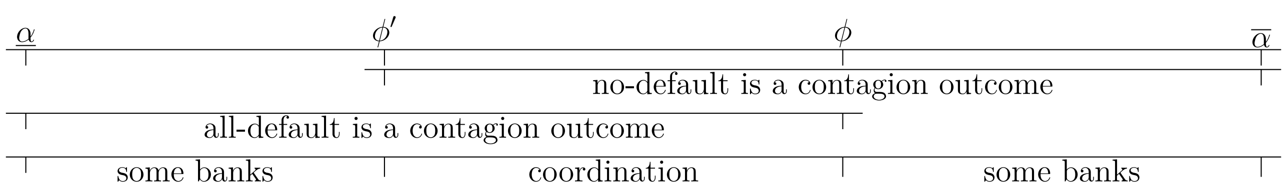

Figure 5: Probability of coordination failures

[11] The exposures are formed by the contracts of bankers. During the contracting stage, banks do not allow the bankers to make insurance promises more than the maximum amount their investments can yield. Then $E_u^{\rightarrow} < (1-s)R_u$. Additionally, bankers would not promise a contractual payment d such that their return falls below zero, so $ds'' < rs'$. Then $s''E_u^{\rightarrow} < s'R_u$.

Note that ϕ'_u is invariant to netting and defaults happen trivially because fundamentals are very severe. Hence, we focus on parametric restrictions on fundamentals, $\underline{\alpha} \geq \phi'$ so that aggregate shocks are not so severe and banks only default because coordination failures.

Condition 1. Bank u does not default if it receives all interbank payments in any contagion outcome: $\underline{\alpha} A_u + R_u - L_u + E_u^{\leftarrow} - E_u^{\rightarrow} \geq 0$. Equivalently, $\underline{\alpha} > \phi'_u$.

This condition requires sufficiently high interconnectedness in the system, through the relevance of $E_u^{\leftarrow} - E_u^{\rightarrow}$. We relegate the explicit restrictions when discussing the matching process across bankers in Section 3.2.4. Under this condition, the region below ϕ' is eliminated and we can easily characterize the probability of a coordination failure as follows,

Lemma 1. *The probability that there is a contagion outcome in which all banks default is given by $F_\alpha(\phi)$. If Condition 1 holds for all banks, $\Phi = F_\alpha(\phi)$.*

If Condition 1 does not hold, our results would be both about financial stability that involves both coordination failures and solvency issues. Our results will focus on $F_\alpha(\phi)$, which is the probability that all banks default in the worst contagion outcome.[12]

A second complication from endogenous exposures is given by the conditional incentives to default on the margin. When the aggregate shock is below the coordination failure cutoff ϕ, any given bank defaulting obtain zero equity, so there are no payments to bankers to be distributed. If the aggregate shock is well above the cutoff ϕ, equity is well above the promises to bankers, and all bankers get paid in full. In contrast, banks close to the defaulting margin may be unable to repay all their bankers after fulfilling the external liabilities to investors, to the creditor, and to other institutions. This complicates bankers' incentives by involving very fine details of the network to appear in expected payoffs. To avoid these technical complications, we further impose the next additional restriction

Condition 2. Interbank assets are larger than internal liabilities: $E_u^{\leftarrow} \geq s' R_u + s''(E_u^{\leftarrow} - E_u^{\rightarrow})$.

Just like Condition 1, we discuss the explicit restrictions on fundamentals that guarantee Condition 2 when discussing the matching structure later.

Lemma 2. *If Condition 2 holds for bank u, all bankers of u get full payment from u whenever there is no coordination failure and* 0 *payment when there is coordination failure.*

If the aggregate shock is around the coordination failure cutoff banks suffer discontinuous losses above and beyond the liquidations. The entire notion of a coordination failure is based on this loss: $E_u^{\leftarrow}$. If these interbank assets are larger than the internal liabilities to the bankers,

[12]Worst equilibrium is well-defined due to supermodularity. See Jackson and Pernoud (2020) for more on multiplicity of equilibria.

$s'R_u + s''(E_u^{\leftarrow} - E_u^{\rightarrow})$, then bankers payoffs are simplified to no payment or full payment, depending solely on whether there is a coordination failure or not. As we will discuss, the coordination failure probability Φ simply scales bankers' payoffs. Jointly with the continuum assumption, we can characterize optimal contracts, as single pairs of bankers can not influence aggregate outcomes.

3.2 Insurance Stage

If both bankers in a pair decide to insure, they sign an optimal contract. We select the Pareto optimal Nash equilibrium, so if at least one banker is strictly better off by insuring, they insure. If both bankers are indifferent, they do not insure.[13] We proceed in three steps to characterize the extent and anatomy of insurance. First we discuss the platform, given a contract and a match, then the optimal contract given a match and finally how the match is formed.

3.2.1 The Platform: OTC or CCP

If the pair i, j has signed a contract, they both observe the signals $\iota_{ij/ji}^{O/C}$ and decide whether to novate the contract, keep it OTC, or annul it. They pick the Pareto dominant option and in case of indifference, they follow these tie breaking rules: annulment, then OTC, and finally CCP.

Even though investors may be willing to finance an average banker, still face the risk of financing a bad banker. Once the banker's quality information is revealed, the investor reassesses the likelihood of a successful investment, and may choose to withdraw the funds at no cost. Formally, if n_i believes that his banker $i \in u$ is a good banker with probability γ_i, then n_i believes that the investment will succeed with unconditional probability

$$\sigma_{i\gamma} \equiv \sigma_{i0} + \gamma_i \left(\sigma_{i1} - \sigma_{i0}\right),$$

in which case n_i obtains rs. Otherwise the investment fails, and n_i gets $rs = 0$. The investor understands, however, that there can be coordination failures, which would reduce the actual payment to a fraction $\mathrm{rr}_{u,\alpha}^L rs$ even if the investment succeeds. Recalling the notation $\omega^* = V(sr)$, the investor's expected payoff from financing is $\sigma_{i\gamma}\Omega_u\omega^*$ where

$$\Omega_u \coloneqq (1 - \Phi) + (1/\omega^*)\Phi\mathbb{E}_\alpha \left[V_I \left(\mathrm{rr}_{u,\alpha}^L sr \right) \Big| \, \alpha < \phi \right] < 1. \tag{6}$$

Here $\Omega_u\omega^* < \omega^*$ due to the possibility of coordination failures.

Definition. We say n_i has *low outside option* if $\omega_i < \sigma_{i0}\Omega_u\omega^*$, *moderate outside option* if $\sigma_{i0}\Omega_u\omega^* < \omega_i < \sigma_{i\gamma}\Omega_u\omega^*$, and *high outside option* if $\sigma_{i\gamma}\Omega_u\omega^* < \omega_i$.

[13] The only case in which both bankers are indifferent is when the optimal contract entails no payments in any state. So it is without loss that they prefer not to insure in this case. Additionally, this is off-the-path of play.

We call $P \in \{O, C\}$ *pursuable for* i if n_i has low outside option or n_i has moderate outside option and the signal generated by the platform is uninformative ($\iota_{ij}^P = 1$). We call $P \in \{O, C\}$ *pursuable for* $\{i, j\}$ if P is pursuable for i and pursuable for j.

Proposition 2. *Consider a matched pair of bankers, and a proposed insurance contract, which is individually rational and yields positive total expected surplus to the pair. The pair keeps the contract OTC if OTC is pursuable for the pair. Otherwise the contract is annulled. Off-the-path of play, n_i withdraws if and only if n_i has moderate outside option and he observes $\iota_{ij} = 0$, or n_i has high outside option.*

For low and high outside options, information about the banker is irrelevant. In the first case the investor does not withdraw and in the second he does. For moderate outside option, information about the banker is relevant because the outside option is low enough to finance an average banker but high enough not to finance a bad banker. Since $\iota_{ij} = 0$ is perfectly revealing of i being a bad banker, withdrawal is strictly dominant if $\iota_{ij} = 0$, and so is annulment by the bankers to conceal the information. Annulment is uniquely Pareto dominant if $\iota_{ij}^O = 0$, which implies that n_i never observes $\iota_{ij}^P = 0$ and n_i never update his beliefs on the path of play.

The pairs write an insurance contract in platform P if and only P is pursuable for the pair. As the contract is individually rational and yields positive total expected surplus, choosing P Pareto dominates annulment when P is pursuable for the pair. When P is not pursuable for the pair, annulment Pareto dominates P as at least one banker loses financing and the other is at best indifferent. CCP is never used because it is always more transparent in the sense that if OTC reveals a banker is bad, CCP also reveals it (this is, $\iota_{ij}^O = 0$ implies $\iota_{ij}^C = 0$). In other words, if $P = C$ is pursuable for the pair, then so is $P = O$. Given our tie-breaking rule, no insurance is ever written though CCP. This is of course a benchmark. When introducing regulation later, CCP would become preferred in certain circumstances.

Proposition 3. *A banker receives funding if and only if his investor has low or moderate outside option. A pair of matched bankers implement an insurance contract if and only if OTC is pursuable for the pair. All implemented contracts are implemented on OTC.*

Figure 6 shows the regions of investor's outside options that leads to financing and insurance. Figure 7 shows, for two bad bankers, the regions of their investors' outside options that lead to specific insurance probabilities. When a banker is good, transparency does not impact insurance probabilities.

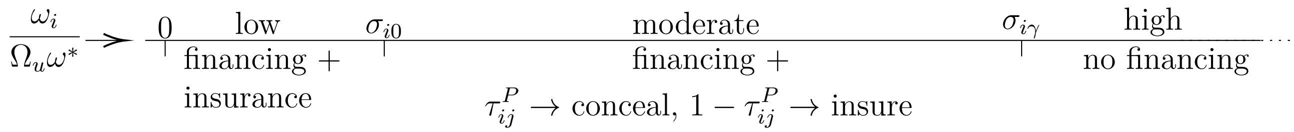

Figure 6: Financing, insurance, and concealing regions of investor outside option for a bad provisional banker early-on-the-line

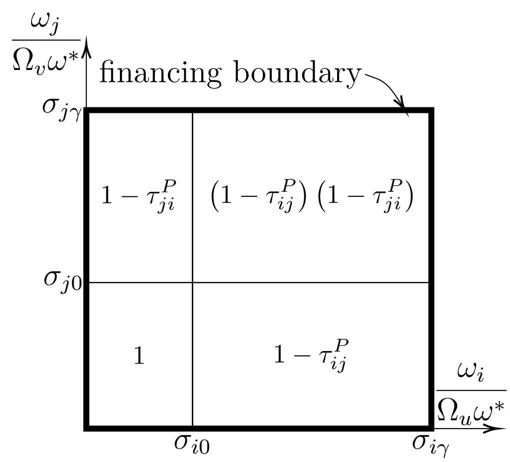

Figure 7: Ex-ante insurance probability between two bad bankers conditional on obtaining funding and getting matched, as a function their investors' outside options

3.2.2 Optimal insurance contract

For a matched pair of bankers, there are insurance gains only in two states of the world; when one investment succeeds and one fails.[14] We denote $d_{ij} = d_{ij}(r, 0) \in \mathbb{R}$ and $d_{ji} = d_{ji}(r, 0) \in \mathbb{R}$, and find the optimal contract $(d_{ij}, d_{ji}) \in \mathbb{R}^2$. Since utility is $V_B(x) = x + \theta \min\{\beta, x\}$ and $rs' > 2\beta$, the optimal risk sharing can be achieved giving both bankers β, which is feasible with only one investments succeeding.

To describe the optimal contract denote

$$\sigma_{ij} = (1-\sigma_i)\sigma_j, \quad \sigma_{ji} = (1-\sigma_j)\sigma_i$$

Here σ_{ij} is the probability that i is "exposed" to j, or j is "liable" to i. This happens when i's investment fails and j's investment succeeds, hence j owes to i. Hence, the expected payoff of banker i who is matched with banker j and signed a contract (d_{ij}, d_{ji}) is given by

[14] Individual rationality can require payments to be made in the state where both investments succeed, not when both investments fail since bankers have nothing to promise. As our focus is clearing and contagion rather than details of contracts we relegate the general case to the appendix and assume $\theta(\sigma_{ji} - \xi) > \sigma_j - \sigma_i$ for all i, j, which no payments are necessary under the state in which both investments succeed.

$$\begin{aligned}
&\Phi\left(\sigma_{ij}V(s''d_{ij}) + \sigma_{ji}V(s'r - s''d_{ij}) + \xi\theta s''(c_u(\mathbb{E}[\alpha|\alpha > \phi] + \zeta_i) - d_{ji})\right)\\
\propto&\underbrace{\sigma_{ij}(s''d_{ij} + \theta\min\{s''d_{ij}, \beta\}) - \sigma_{ji}s''d_{ji}}_{\text{insurance gain}} + \theta\min\{s'r - s''d_{ij}, \beta\}) - \underbrace{\xi\theta s''d_{ij}}_{\text{coll. cost}}\\
\propto&\sigma_{ij}\min\{d_{ij}, \kappa\} + \sigma_{ji}\min\{rs'/s'' - d_{ji}, \kappa\}) - d_{ji}\xi + (\sigma_{ij}d_{ij} - \sigma_{ji}d_{ji})/\theta
\end{aligned}$$

Proposition 4. *The unique optimal contract is given by* $d_{ij} = d_{ji} = \kappa$*. Total expected contracting gain is* $(\sigma_{ij} + \sigma_{ji} - 2\xi)\beta\theta$*.*

Consider the state when i's investment fails and j's investment succeeds. By promising d_{ij}, j incurs utility cost $d_{ij}s''$ with a utility gain to i being $s''d_{ij} + \theta\min\{\beta, s''d_{ij}\}$. The total insurance gain is then $\theta\min\{\beta, s''d_{ij}\}$ which is maximized by setting $d_{ij} \geq \kappa \equiv \frac{\beta}{s''}$. Similarly, $d_{ji} \geq \kappa$, hence the optimal contract is (κ, κ), also individual rational. Figure 8 shows indifference curves (for sum of expected payoffs), individual rationality constraints, and the optimal contract.

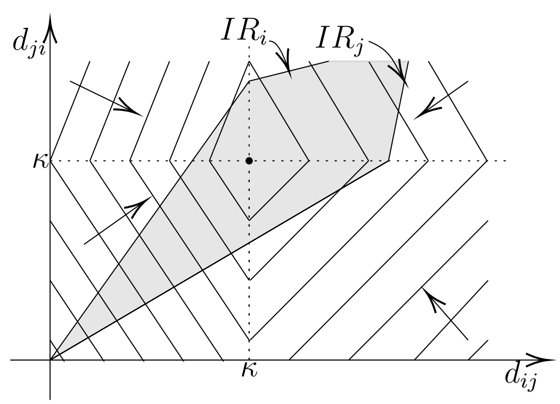

Figure 8: Indifference curves for the sum of expected payoffs and IR constraints

Figure 8 shows the individual rationality sets (how much an individuals is willing to pay conditional on the payment received from the other agent) and the direction of utilities, until the agent obtains κ, which guarantees consuming β, the utility of the agent increases with more insurance, but after that level the agent is satisfied (no more utility obtained from insurance) but still faces a collateral cost. In the figure the optimal insurance contract is feasible, which is not always the case. Individual rationality can be violated when success probabilities are significantly uneven. For example, if $\sigma_{ij} - \sigma_{ji}$ is sufficiently large, the expected transfer to j is significantly smaller than the expected transfer to i. This is explored in the appendix.

We can now compute the ex-ante probability that an insurance contract is written. The *pursuance rate* π_i is the ex-ante probability of pursuing a contract, defined as

$$\pi_i(\cdot) \coloneqq \frac{\sigma_{i0} + (1-\cdot)\gamma_i(\sigma_{i1} - \sigma_{i0})}{\sigma_{i0} + \gamma_i(\sigma_{i1} - \sigma_{i0})}$$

While a platform is always pursuable if the banker is good ($q_i = 1$), conditional on being bad ($q_i = 0$), the ex-ante *pursuance rate* depends on the probability τ_{ij}^P that the platform generates a signal about it. Succinctly, $\pi_i((1-q_i)\tau_{ij}^P)$ is the ex-ante pursuance rate of P for i. We denote

$$\pi_{Pij} \coloneqq \pi_i((1-q_i)\tau_{ij}^P) \times \pi_j((1-q_j)\tau_{ij}^P)$$

Proposition 5. *The ex-ante probability of an implemented insurance contract between i and j, if matched, is π_{Oij}.*

3.2.3 Model of banker matching

Here we describe the matching process. We assume bankers within a bank do not match, so we focus on undiversified portfolios and the network structure. As we work with matching between continuums of bankers, we define the masses of actually matched bankers directly, rather than defining matching probabilities that would generate the corresponding masses of matched bankers. Our method is akin to stochastic block models and graphons, adapted to one-to-one matchings. See Erol et al. (2020) for details.

Recall that γ_i is the probability that banker i is a good banker. Denote $\gamma_{iq} = 1 - \gamma_i + 2q\gamma_i$ the probability that i has quality q and $\gamma_{uq} = \mu_u^{-1} \int_u \gamma_{iq} \mathsf{d}i$ the probability that a random banker in u has quality q. Note $\gamma_{u0} + \gamma_{u1} = 1$. These are given outside of the matching structure.

Definition. A *matching structure* is a vector of non-negative numbers $M = ((\mu_{uv}^{qq'})_{v,q,q'}, (\mu_{u0}^{q})_q)_u$ that satisfies

- (*Measure Preservation*) $\mu_{uv}^{qq'} = \mu_{vu}^{q'q}$ for all u, v, q, q', and
- (*Consistency*) $\mu_{u0}^{q} = \gamma_{uq}\mu_u - \sum_{v'} \sum_{q''} \mu_{uv'}^{qq''} \geq 0$ for all u, q.

A *matching* drawn ex-ante from M is a one-to-one measure preserving mapping between bankers such that for all u, v, q, q', there is $\mu_{uv}^{qq'}$ mass of bankers in u with quality q that are matched with a banker in v with quality q', and there is μ_{u0}^{q} mass of unmatched bankers of quality q in bank u.

There is $\mu_{uv}^{qq'}$ mass of bankers in u with quality q that are matched with a banker in v with quality q'. If this is counted from the side of bank v, the mass of bankers in v with quality q' that are matched with a banker in u with quality q is $\mu_{vu}^{q'q}$. The first condition $\mu_{uv}^{qq'} = \mu_{vu}^{q'q}$ ensures that the resulting one-to-one matching between these bankers in u with quality q and in v with quality v' is a measure preserving mapping. The mass of bankers in u with quality q that have a match is then given by $\sum_{v'} \sum_{q''} \mu_{uv'}^{qq''}$. The remaining bankers in u with quality q are unmatched, which has a mass $\gamma_{uq}\mu_u - \sum_{v'}\sum_{q''}\mu_{uv'}^{qq''}$. The consistency condition simply means that $\mu_{u0}^q \geq 0$ represents the mass of unmatched bankers in u who have quality q.

Given a matching structure $M = ((\mu_{uv}^{qq'})_{v,q,q'}, (\mu_{u0}^q)_q)_u$, the following protocol generates a matching drawn from M wherein each banker's match is independent of his identity.

The protocol. The process takes each of $\gamma_{uq}\mu_u$ mass of bankers in u with quality q and distribute them to "matching categories." The process assigns each banker among this $\gamma_{uq}\mu_u$ mass independently to the *matching category* $\{(q,u),(q',v)\}$ with probability $\mu_{uv}^{qq'}(\mu_u\gamma_{uq})^{-1}$, and to the matching category Null with the residual probability $1 - \sum_v \sum \mu_{uv}^{qq'}(\mu_u\gamma_{uq})_{q'}^{-1}$. By the Consistency property of M, the probability of being assigned to Null is equal to $\mu_u^{-1}\mu_{u0}^q$. Once all allocations to matching categories are made, consider each category $\{(q,u),(q',v)\}$. The mass of bankers from u who are allocated to category $\{(q,u),(q',v)\}$ is $(\mu_u\gamma_{uq})\,\mu_{uv}^{qq'}\,(\mu_u\gamma_{uq})^{-1} = \mu_{uv}^{qq'}$. Note that by construction, all of these bankers have quality q. Similarly, the mass of bankers from v who are allocated to the category $\{(q,u),(q',v)\}$ is $\mu_{vu}^{q'q}$. These bankers have quality q'. By the Measure Preservation property of M, $\mu_{uv}^{qq'} = \mu_{vu}^{q'q}$, and so there are equal masses of bankers from u and v in category $\{(q,u),(q',v)\}$. Take each banker in the category from bank u and allocate an index drawn i.i.d. from uniform $U[0,\mu_{uv}^{qq'}]$. Do the same for bankers in the category from v. Then, match bankers on two sides that have the same index. Finally, leave all bankers in category Null unmatched. As being assigned to Null has probability $\mu_u^{-1}\mu_{u0}^q$, the mass of unmatched bankers of quality q in u is μ_{u0}^q. Notice that banker indices do not appear in the process, and the resulting matching is measure-preserving since there are finitely many categories.

Notice that the identity of banker i influences his match only through his bank's characteristics and his realized quality, not the name i in and of itself. Then, for any two bankers $i \in u$ and $j \in v$ of quality q and q', the probability density of i and j getting matched is

$$\tilde{\mu}_{uv}^{qq'} := \frac{\mu_{uv}^{qq'}}{\mu_u\gamma_{uq}}\frac{\mu_{uv}^{qq'}}{\mu_v\gamma_{vq'}}\frac{1}{\mu_{uv}^{qq'}} = \frac{\mu_{uv}^{qq'}}{\mu_u\gamma_{uq}\mu_v\gamma_{vq'}}$$

Bankers' insurance opportunities are determined by the banks' identities. This captures the idea that banks serve as hubs of communication across bankers in the economy and reduce search and matching frictions.

3.2.4 Consolidation and equilibrium

The system results from aggregating the above-mentioned variables. Optimal contracts have been stated for given success probabilities σ_i, σ_j in Proposition 4 but σ_i, σ_j depend on banker qualities. In finding aggregate exposures, we need to take into account the dependence of success probabilities on the random banker quality and the probability of qualities. Accordingly, index all relevant variables and functions with banker quality.

Denote $\pi_u^0 = \int_u \pi_i(d)\mathtt{d}i$ the average success rate by bankers in u, and the *exposure scaler*[15]

$$e_{uv}^* = \int_{i\in u}\int_{j\in v}\sum_{q,q'}\gamma_{iq}\gamma_{jq'}\tilde{\mu}_{uv}^{qq'}\pi_{Oij}^{qq'}(\sigma_{jq'}-\sigma_{iq})\kappa\mathtt{d}j\mathtt{d}i$$

The exposure scaler e_{uv}^* reflects the critical ingredients of the model except yet the novation decisions. For a pair of bankers i, j from banks u, v, their qualities are drawn first. Then they get matched with each other with probability mass $\tilde{\mu}_{uv}^{qq'}$. There is $\pi_{Oij}^{qq'}$ probability that they implement a contract. The contracting probability is influenced by the generation of signals (the level of transparency). Banker qualities affect the rate at which the contract is pursued because good bankers are not concerned with investor information and do not conceal information, so always engage in writing the insurance contract. Finally, the term $(\sigma_{jq'}-\sigma_{iq})\kappa \equiv (\sigma_{ij}^{qq'}-\sigma_{ji}^{q'q})\kappa \equiv D_{ij}^{qq'} - D_{ji}^{q'q}$ is the expected infinitesimal net liability of v to u due to the contract between i and j. Once all of these factors are integrated over all banker pairs, the resulting gross liability of v to u is determined.

Proposition 6. *The equilibrium system S^* is given by*[16]

$$E'_{uv} = \langle e_{uv}^*\rangle\,,\;\; E'_{vu} = \langle e_{vu}^*\rangle\,,\;\; E'_{uC} = E'_{Cu} = 0$$
$$A_u = \mu_u c_u,\;\; R_u = \mu_u \pi_u^0 r_u,\;\; L_u = \mu_u(s\pi_u^0 r_u + m)$$

The equilibrium coordination failure cutoff is

$$\phi^* = \min_u c_u^{-1}\Big(\mu_u^{-1}\sum_v \langle e_{vu}^*\rangle + m^*\Big)$$
$$\textit{where } m_u^* = m - \pi_u^0(1-s)r$$

Going forward we assume $m_u^* > 0$ to capture the riskiness of projects and their role in triggering coordination failures. Low m^* means that the coordination failure cutoff could never be crossed regardless of the aggregate shock. Then higher size collateral (c_u) and higher investment returns ($\pi_u^0 r$) reduce the probability of coordination failures by providing more assets as buffers. Higher interbank exposure (e_{uv}^*), higher payments to investors (s), higher payments to creditors (m) increase the probability by increasing liabilities.

Regarding the effect of transparency on the probability of coordination failure, the only terms that depend on the level of transparency are exposure scalers e_{uv}^*, through the pursuance rate π_{Oij}, which is decreasing in transparency.

While gross exposures are decreasing in transparency, net liabilities $D_{ij}^{q_i q_j} - D_{ji}^{q_j q_i}$ can be

[15] To guarantee Conditions 1 and 2 for all following results, we assume $\underline{\alpha} c_u < \frac{1}{\mu_v}\sum_v e_{vu}^* + m - (1-s)\pi_u^0 r$ and $s'\pi_u^0 r + s''\sum_v e_{vu}^* \leq \sum_v \langle e_{uv}^*\rangle$.

[16] Notice that $e_{uv}^* + e_{vu}^* = 0$.

positive or negative, so the net effect of transparency is ambiguous. Still, if both gross are decreasing, also will be the net as total liabilities decline.[17] This is, unless there are large asymmetries is banker quality probabilities and success probabilities of bankers across different banks. The effect of netting is also observable as follows. Denote

$$e_u^{\rightarrow} = \sum_v \langle e_{vu}^* \rangle \,, \quad e_u^{\leftarrow} = \sum_v \langle e_{uv}^* \rangle$$

Note that $e_{uv}^* + e_{vu}^* = 0$, and $e_u^{\rightarrow} - e_u^{\leftarrow} = \sum_v e_{vu}^*$. When there is a coordination failure, the exposure to u increases from $\sum_v e_{vu}^*$ to $e_u^{\rightarrow}$ as the interbank assets $e_u^{\leftarrow}$ are wiped in a coordination failure, reducing the self-fulfilling contagion cutoff to ϕ^*. In a mechanical sense, if there is more netting keeping gross exposures fixed, $e_u^{\rightarrow}$ and $e_u^{\leftarrow}$ would decrease while keeping $e_u^{\rightarrow} - e_u^{\leftarrow}$ fixed. This would reduce interbank asset cost of coordination failures ($e_u^{\leftarrow}$) and self-fulfillingly make the coordination failures less likely ($e_u^{\rightarrow}$).

4 Characterization with Capital Requirements

Here, we want to understand how the system, the equilibrium, and the chances of contagion endogenously change when *capital requirements* are imposed on a bank's aggregate positions, which is the aggregation of the contracts, so each banker needs to follow internal regulation imposed by his bank that ensures the bank's capital adequacy.

4.1 Capital requirements

We follow current regulation and assume that each bank must maintain a capital adequacy ratio, defined as the ratio of their regulatory capital to risk-weighted assets.[18]

The regulatory capital REG_u (this is, the capital subject to regulation) is given by

$$\text{REG}_u = \int_u (c_u(\zeta_i + \mathbb{E}[\alpha]) - m)\mathtt{d}i + (1-s)r \int_{u_R} \sigma_i \mathtt{d}i$$

The insurance contracts do not go into regulatory capital as they are off-balance sheet items.

Asset classes are publicly known ex-ante: projects face risk α, investments of each banker faces risk $(\sigma_{i0}, \sigma_{i1})$, and contractual interbank exposures face risk $(\sigma_{i0}, \sigma_{i1}, \sigma_{j0}, \sigma_{j1})$. Accordingly, risk weights are given by RW_A for projects, RW_R^i for i's investment class, $\text{RW}_{C/O}^{ij}$ for the class of insurance contract that insures against returns from investments classes of i and j, for example

[17] To fix ideas, if one assumes that all bankers are ex-ante identical, then some algebra shows that $e_{uv}^* = (\eta_{uv} - \eta_{vu})\pi_O^{01}$. Total exposures to u are $\propto \sum_v \langle \eta_{uv} - \eta_{vu} \rangle \pi_O^{01}$ which is decreasing in transparency $\tau_{ij}^O \equiv \tau^O$.

[18] See Consolidated Reports of Condition and Income for a Bank with Domestic and Foreign Offices—FFIEC 031 form for details on how regulatory capital and risk weighted assets are calculated. Available at https://www.ffiec.gov/pdf/FFIEC_forms/FFIEC031_202112_f.pdf

an exotic derivative. The regulation differentiates between cleared and OTC exposures, so RW_C^{ij} applies to cleared exposures and RW_O^{ij} applies OTC exposures, where $\text{RW}_O^{ij} \geq \text{RW}_C^{ij}$. It is important to note that risk weights for (derivative) insurance contracts apply to net asset contracts, not the net liability contracts.[19]

Let u_O be the set of insured bankers in u that kept contracts OTC, and u_C be those that novated. Let i^* denote the matched banker j of banker i (so to integrate only over bank u's bankers). Then the risk weighted assets RWA_u of bank u at the time regulation is given by

$$\begin{aligned}\text{RWA}_u = {} & \text{RW}_A c_u \int_{u_R} (\zeta_i + \mathbb{E}[\alpha])\mathtt{d}i + r \int_{u_R} \text{RW}_R^i \sigma_i \mathtt{d}i \\ & + \int_{u_O} \text{RW}_O^{ii^*} \langle \mathbb{E}\left[d_{ii^*} - d_{i^*i}\right] \rangle \, \mathtt{d}i + \int_{u_C} \text{RW}_C^{ii^*} \langle \mathbb{E}\left[d_{ii^*} - d_{i^*i}\right] \rangle \, \mathtt{d}i\end{aligned}$$

The capital requirements imply a certain capital adequacy ratio, CAR_u. This is, the regulatory capital of u must be at least CAR_u fraction of the risk weighted assets of u:

$$\text{REG}_u \geq \text{CAR}_u \times \text{RWA}_u$$

The bank must ensure its capital adequacy by internally regulating its (infinitesimal) bankers. This is typically done by risk divisions of banks by putting restrictions on traders through automated algorithms. In our framework, bank u imposes every banker to uphold an *internal capital constraint* that is the parallel reduction of the capital requirement to individual bankers. We call this the *internal regulation*. For a banker i who wants to implement an insurance contract with a matched banker that we denote i^*, internal capital constraint is

$$(\zeta_i + \mathbb{E}[\alpha])\, c_u - m + (1-s) r\sigma_i \geq \text{CAR}_u \left(\text{RW}_A(\zeta_i + \mathbb{E}[\alpha])c_u + \text{RW}_R^i r\sigma_i + \text{RW}_{O/C}^{ii^*} \langle \mathbb{E}\left[d_{ii^*} - d_{i^*i}\right] \rangle \right)$$

Here $\text{RW}_{O/C}^{ii^*}$ depends on whether the pair i, i^* decide to novate their contract or not.

For an arbitrary banker i, define $\psi_A^i = 1 - \text{CAR}_{b_i}\text{RW}_A$, $\psi_R^i = 1 - s - \text{CAR}_{b_i}\text{RW}_R^i$, and $\rho_P^{ij} = \text{CAR}_{b_i}\text{RW}_P^{ij}$ for $P \in \{O, C\}$. The *net regulatory capital* of banker i can be written as

$$k_i \equiv \psi_R^i \sigma_i r + \left(\psi_A^i(\zeta_i + \mathbb{E}[\alpha])\right) c_{b_i} - m,$$

which we assume always positive.[20] The internal capital constraint that i faces is then given by

$$k_i \geq \rho_P^{ii^*}(D_{ii^*} - D_{i^*i}) \tag{7}$$

[19]See Consolidated Reports of Condition and Income for a Bank with Domestic and Foreign Offices—FFIEC 031 form (at https://www.ffiec.gov/pdf/FFIEC_forms/FFIEC031_202112_f.pdf). Pages 78 items 20 and 21 require reporting the positive part of the fair value of cleared and OTC derivatives in risk weighted assets.

[20]The internal capital constraint for bankers without insurance contracts is $k_i \geq 0$.

Importantly, $\rho_O^{ii^*} \geq \rho_C^{ii^*}$ meaning the OTC regulation is tighter than CCP regulation.

Novation and insurance under capital constraints. The contracts must uphold the capital adequacy constraint for the chosen platform. Given a contract, we say the choice $P \in \{O, C\}$ is *adequate* if the contract respects inequality (7) for the platform P. Given a contract, the pair either annuls the contract, or chooses an adequate option. As before, they are assumed to pick the Pareto dominant option, with the same priority ranking in case of equivalence.

Given this continuation regarding novation, optimal contracts are defined as before: individually rational expected payoff maximizing contracts. As before, banker play the Pareto optimal Nash equilibrium.

4.2 Optimal contract with capital constraints

Capital regulation differs across platforms, so there are two contracts to consider: OTC-optimal and CCP-optimal, which differ by the constraint they face. OTC-optimal contracts display *smaller insurance gains* as capital regulation is more binding relative to CCP-optimal contracts. OTC-optimal contracts, however, display *larger pursuance rates* as their are more opaque relative to CCP-optimal contracts. OTC-optimal contracts. Since the platform has to be chosen before signals are released, bankers will choose the platform that yields a higher expected payoff from resolving this regulation-opacity trade-off.

Proposition 7. *Consider a matched pair $i \in u$, $j \in v$. Denote*

$$\xi_{ij}^* = \frac{\sigma_{ij}\sigma_{ji}}{\sigma_{ij} + \sigma_{ji}}$$

The unique P-optimal contract is determined as follows. If cost of collateral is low, $\xi < \xi_{ij}^*$, then

$$D_{Pij} = \sigma_{ij} d_{Pij} = \max\left\{\kappa\sigma_{ij}\, ,\, \kappa\sigma_{ji} - k_j \big/ \rho_P^{ji}\right\}$$

If cost of collateral is high, $\xi > \xi_{ij}^*$, then

$$D_{Pij} = \sigma_{ij} d_{Pij} = \min\left\{\kappa\sigma_{ij}\, ,\, \kappa\sigma_{ji} + k_i \big/ \rho_P^{ji}\right\}.$$

We explain Proposition 7 using Figure 9. Without loss, suppose that $\sigma_{ij} \geq \sigma_{ji}$. In the unconstraint optimal contract (κ, κ), the banker with lower success probability, i, gets paid more often. He brings his bank positive expected payout, which must be less than his net regulatory capital k_i. If $\kappa\sigma_{ij} - \kappa\sigma_{ji} > k_i/\rho^{ij}$, i does not have adequate regulatory capital for the unconstraint optimal contract. Then i must pay j more in the contract, $D_{ji} > \kappa$, to increase liabilities and reduce net assets of i down to a compliant level, or i must be paid less by j, $D_{ij} < \kappa$, to achieve the same. Which option is optimal depends on the cost of collateral. If i

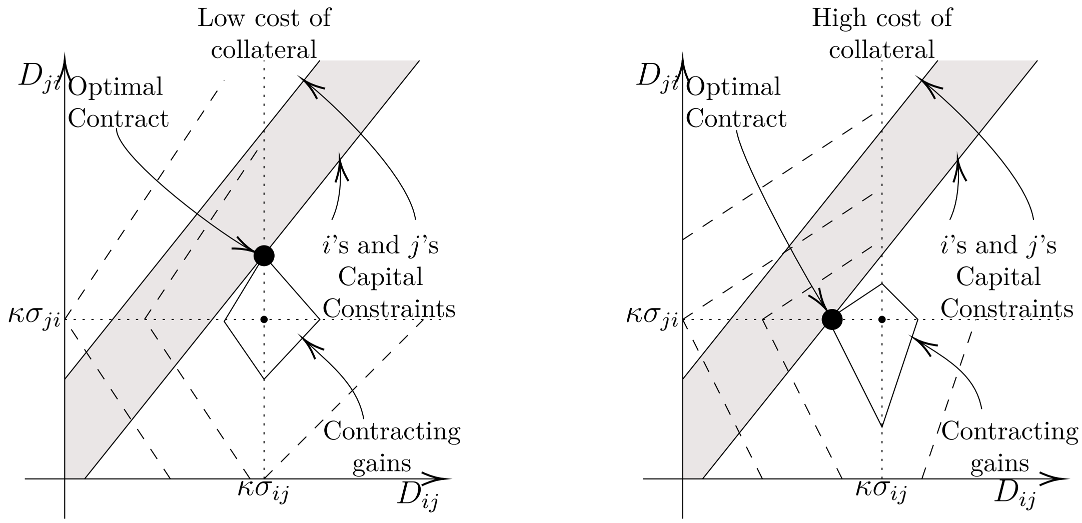

Figure 9: Optimal contract when capital constraint binds

pays more, this addition keeps insurance gains at the maximum level by making $D_{ij} = \kappa\sigma_{ij}$ and $D_{ji} = \kappa\sigma_{ij} - k_i/\rho^{ij} > \kappa\sigma_{ji}$, but comes at the additional cost of collateral. This is optimal when the cost of collateral is low. If i gets paid less, insurance gains are lower but saves on collateral costs by making $D_{ij} = \kappa\sigma_{ji} + k_i/\rho^{ij} < \kappa\sigma_{ij}$ and $D_{ji} = \kappa\sigma_{ji}$. This is preferred when the cost of collateral is high. Proposition 7 states these jointly for both cases of $\sigma_{ji} \leq \sigma_{ij}$ and $\sigma_{ji} \geq \sigma_{ij}$.[21]

In what follows we assume low cost of collateral $\xi < \xi^*_{ij}$ for simplicity, as insurance is not distorted in this case. Total insurance gains are maximal but collateral costs are varied, so the internal regulation for the banker with lower success probability may bind the pair.

4.3 Novation and ex-ante probabilities of insurance and financing

The insurance gains from OTC-optimal contracts are smaller than those from CCP-optimal contracts only if capital constraints bind. If they do not, i.e. $d_O \equiv \kappa$, then $D_O = D_C = (\kappa\sigma_{ij}, \kappa\sigma_{ji})$, and so total insurance gains are the same, bankers sign $d \equiv \kappa$ and keep it OTC. When the OTC constraints bind, i.e. $d_O \not\equiv \kappa$, then OTC is not adequate for the CCP-optimal contract and d_C can be implemented only on CCP, when CCP is pursuable, which happens with probability π_{Cij}. On the other hand, the inferior OTC-optimal contract can be pursued on either platform with higher probability π_{Oij} because of opacity, which value can be defined as

Definition. The *value of opacity* for i and j is the difference in expected total insurance gain, net of collateral costs, between an OTC-optimal contract and a CCP-optimal contract.

[21]Note that the described contracts do not violate the incentive constraints because the rate of substitution between expected payments to satisfy capital constraints is 1, whereas the same rate for incentive constraint is distorted by costs of collateral making it larger than 1.

Lemma 3. *Consider a matched pair $i \in u$, $j \in v$. The value of opacity (net of collateral costs) for the pair is Θ_{ij} where*

$$\begin{aligned}\Theta_{ij} &= \overbrace{(\pi_{Oij})\left(\beta(\sigma_{ij}+\sigma_{ji}) - \xi s''\left(d_{Oij}+d_{Oji}\right)\right)}^{\text{OTC contracting gains}} - \overbrace{\pi_{Cij}\left(\beta(\sigma_{ij}+\sigma_{ji}) - \xi s''\left(d_{Cij}+d_{Cji}\right)\right)}^{\text{CCP contracting gains}} \\ &= \underbrace{(\pi_{Oij}-\pi_{Cij})\left(\sigma_{ij}+\sigma_{ji}\right)\beta}_{\text{insurance gains}} - \underbrace{\xi s''\left(\pi_{Oij}\left(d_{Oij}+d_{Oji}\right) - \pi_{Cij}\left(d_{Cij}+d_{Cji}\right)\right)}_{\text{cost of collateral}}\end{aligned}$$

Conditional on signing a contract, the pair signs the OTC-optimal contract if the value of opacity is positive, and the CCP-optimal contract if the value of opacity is negative.

The next Proposition summarizes the optimal platform used for insurance contracts.

Proposition 8. *Consider a matched pair $i \in u$, $j \in v$.*

A CCP-optimal contract is implemented on CCP if and only if CCP is pursuable and the value of opacity is negative, which happens with ex-ante probability π_{Cij}.

An OTC-optimal contract is implemented on OTC if and only if OTC is pursuable and the value of opacity is positive, which happens with ex-ante probability π_{Oij}.

Otherwise no contract is implemented.

4.4 Collateral and the value of opacity

The sign of the value of opacity net of collateral costs for a pair is the sole determinant of whether the pair chooses to novate or not. In trying to shift exposures to central clearing, the regulator then could try to tighten capital constraints of OTC relative to CCP to increases collateral costs and shift the value of opacity from positive to negative in OTC contracts. Consider the following ranges of collateral constraints,

Definition. Take a matched pair i, j and let i be the one with lower success probability: $\sigma_i \leq \sigma_j$. Assume capital constraints are more stringent for OTC contracts, this is $\rho_O^{ij} > \rho_C^{ij}$. The pair is said to have *high collateral* if $\rho_O^{ij}(\sigma_j - \sigma_i)\kappa < k_i$, *medium collateral* if $\rho_C^{ij}(\sigma_j - \sigma_i)\kappa < k_i < \rho_O^{ij}(\sigma_j - \sigma_i)\kappa$, and *low collateral* if $k_i < \rho_C^{ij}(\sigma_j - \sigma_i)\kappa$.

Proposition 9. *Consider a matched pair i, j and suppose that both platforms are pursuable for the pair. Without loss take $\sigma_i \leq \sigma_j$. Then $D_{Pij} = \kappa\sigma_{ij}$ for both $P \in \{O, C\}$.*

If the pair has high collateral, then neither OTC nor CCP constraints bind, $D_{Oji} = \kappa\sigma_{ji}$, and $D_{Cji} = \kappa\sigma_{ji}$. The contract is never novated as $\Theta_{ij} > 0$.

If the pair has medium collateral, OTC constraint binds, CCP constraint does not, $D_{Oji} = \kappa\sigma_{ij} - k_i/\rho_O^{ij}$, and $D_{Cji} = \kappa\sigma_{ji}$. The contract is novated (this is $\Theta_{ij} < 0$) if and only if, .

$$k_i < \widetilde{N}_{ij} := \frac{\rho_O^{ij}}{\pi_{Oij}}\left(\pi_{Cij}\left(\sigma_{ij}-\sigma_{ji}\right) - \left(\pi_{Oij}-\pi_{Cij}\right)\left(\sigma_{ij}+\sigma_{ji}\right)\left(\frac{\sigma_{ji}}{\xi}-1\right)\right)\kappa. \tag{8}$$

If the pair has low collateral, then both OTC and CCP constraints bind, $D_{Oji} = \kappa\sigma_{ij} - k_i/\rho_O^{ij}$*, and* $D_{Cji} = \kappa\sigma_{ij} - k_i/\rho_C^{ij}$*. The contract is novated (this is* $\Theta_{ij} < 0$*) if and only if,*

$$k_i > \underset{\sim}{N}_{ij} := \max\left\{0, \frac{\pi_{Cij}}{\rho_C^{ij}} - \frac{\pi_{Oij}}{\rho_O^{ij}}\right\}^{-1} (\pi_{Oij} - \pi_{Cij})(\sigma_{ij} + \sigma_{ji})\left(\frac{\sigma_{ji}}{\xi} - 1\right)\kappa. \tag{9}$$

This proposition is, without loss of generality, written for the case $\sigma_i \leq \sigma_j$, as Proposition 7. In this case j is more likely to succeed and pay to i. Since i has a net exposure to j she has to hold capital, but j does not. Hence $D_{Pij} = \kappa\sigma_{ij}$ regardless of regulation or platform, but $D_{Pji} = \max\{\kappa\sigma_{ji}, \kappa\sigma_{ij} - k_i(\rho_P^{ij})^{-1}\}$ since i has to potentially distort the contract given the collateral cost it implies.

In the case of high collateral the OTC constraint does not bind, hence the CCP constraint does not bind either. Then $d \equiv \kappa$, the pair uses OTC regardless, and regulation is ineffective in shifting exposures. Bankers keep the contract OTC, as it is more pursuable given its opacity.

In the case of medium collateral, the pair entertains switching from a constrained OTC-optimal contract with opacity to an unconstrained CCP-optimal contract with transparency. The benefit from novating the contract comes from savings in collateral, which are more relevant for i if not having that much collateral, k_i, as given by condition (8). Hence, conditional on a region of medium collateral, as regulation gets tighter, bankers with higher collateral among the medium collateral bankers are the last to switch to CCP.

In the case of low collateral, both CCP and OTC capital contraints bind. In this case bankers compare the opaque OTC-optimal optimal contract with the transparent CCP-optimal contract. The gains from relaxing regulation by shifting to CCP is given by the relative degree of pursuance of the platform relative to its regulatory cost $\frac{\pi_{Pij}}{\rho_P^{ij}}$ as in equation (9). If this ratio is lower for CCP than for OTC, and the lower regulation does not compensate the loss from transparency, the pair never novates. If regulation become more stringent for OTC (say an increase of ρ_O^{ij} or reduction of ρ_C^{ij}) those bankers with high collateral will novate first. Hence, conditional on a region of low collateral, as regulation gets tighter, bankers with lower collateral among the low collateral bankers are the last to switch to CCP.

4.5 Equilibrium system

Denote $\{\{\cdot\}\}$ the Iverson bracket. This is, $\{\{x\}\} = 1$ if x holds and $\{\{x\}\} = 0$ otherwise. Out of the contract between i and j, the resulting (infinitesimal) gross exposure of u to v is $D_{Oij}\{\{\Theta_{ij} > 0\}\}$. The resulting gross exposure of u to CCP is $D_{Cij}\{\{\Theta_{ij} < 0\}\}$. Indexing all relevant variables and functions by the quality of bankers we obtain the following proposition.

Proposition 10. *The equilibrium system is given by*

$$E'^{*}_{vu} = \langle e^{**}_{vu} \rangle\,,\;\; E'^{*}_{uv} = \langle e^{**}_{uv} \rangle\,,\;\; E'^{*}_{Cu} = \langle \sum_{v} e^{**}_{vCu} \rangle,\;\; E'^{*}_{uC} = \langle \sum_{v} e^{**}_{uCv} \rangle$$

$$A_u = \mu_u c_u,\;\; R_u = \mu_u \pi^0_u r_u,\;\; L_u = \mu_u (s\pi^0_u r_u + m)$$

where

$$e^{**}_{uv} := \sum_{q,q'} \int_{i\in u} \int_{j\in v} \pi^{qq'}_{ijO} (D^{qq'}_{Oij} - D^{q'q}_{Oji}) \{\{\Theta^{q_i q_j}_{ij} > 0\}\} \gamma_{iq}\gamma_{jq'} \tilde{\mu}^{qq'}_{uv} \,\mathtt{d}j\,\mathtt{d}i$$

$$e^{**}_{uCv} := \sum_{q,q'} \int_{i\in u} \int_{j\in v} \pi^{qq'}_{ijC} (D^{qq'}_{Cij} - D^{q'q}_{Cji}) \{\{\Theta^{q_i q_j}_{ij} < 0\}\} \gamma_{iq}\gamma_{jq'} \tilde{\mu}^{qq'}_{uv} \,\mathtt{d}j\,\mathtt{d}i$$

for all u, v. Define exposure-to-u scaler as

$$e^{\Rightarrow}_{u} = \sum_{v} \langle e^{**}_{vu} \rangle + \langle \sum_{v} e^{**}_{vCu} \rangle$$

The equilibrium probability of a coordination failure is $\Phi^* = F_\alpha(\phi^*)$ where

$$\phi^* = \min_u c_u^{-1} \Big(\mu_u^{-1} e^{\Rightarrow}_{u} + m^*_u \Big)$$

Notice that exposures e^{**}_{uv} and e^{**}_{uCv}, and ultimately the coordination failure Φ^* cna be written in closed form by substituting D from Proposition 7 and $\{\{\Theta^{q_i q_j}_{ij} > 0\}\}$ from Proposition 9.

5 Characterization when banks just differ on collateral

Assume that all bankers are identical, this means $\sigma_{i0}, \sigma_{i1}, \gamma_i, \tau^O_{ij}, \tau^C_{ij}, RW^i_R, RW^i_A, RW^{ij}_{O/C}$ are identical across all bankers and pairs. Also assume that banks have the same capital adequacy ratio CAR_u. There is only heterogeneity on bank's sizes and value of the projects, this is both (c_u and μ_u). Though differences in size, this also has implications for matching.

When bankers in a match are of the same quality (this is $q_i = q_j$), $\sigma_{ij} = \sigma_{ji}$. In these cases capital constraints do not bind and the pair signs (κ, κ) always, without contributing to bilaterally netted exposures between banks. If one banker in the match is good, however, and the other is bad, the pair adds to bilateral exposure between banks. Accordingly, the relevant variables for the coordination failure are those that relate to pairs of bankers of opposite quality. Next, we drop banker indices, refine notation, and introduce some simplifying definitions for brevity and easier interpretation.

For a match with a good and a bad banker, denote[22]

$$\Delta \coloneqq (\sigma_1 + \sigma_0 - 2\sigma_0\sigma_1)\left(\sigma_0(1-\sigma_1)/\xi - 1\right)\kappa, \qquad \delta \coloneqq (\sigma_1 - \sigma_0)\kappa$$
$$\overline{k}_u \coloneqq \psi_R\sigma_0 + (\psi_A(\mathbb{E}[\alpha] + Z) - m)c_u, \qquad \underline{k}_u \coloneqq \psi_R\sigma_0 + (\psi_A(\mathbb{E}[\alpha] - Z) - m)c_u$$

5.1 The novation gap and the opacity gap.

Using this simplified notation, we can define the opacity and novation gaps, and the range of parameters for which there is novation, as follows

Definition. We call Γ_{OP} the *opacity gap,* Γ_{CR} the *(capital) regulation gap, given by*

$$\Gamma_{OP} = 1 - \frac{\pi(\tau^C)}{\pi(\tau^O)}, \qquad \Gamma_{CR} = 1 - \frac{\rho_C}{\rho_O}$$

We call $\overline{N}$ the *upper novation cutoff* and $\underline{N}$ lower novation cutoff, given by[23]

$$\overline{N} = \rho_O\left(\delta - \Gamma_{\mathrm{OP}}(\Delta + \delta)\right), \qquad \underline{N} = \rho_C \max\left\{0, \Gamma_{\mathrm{CR}} - \Gamma_{\mathrm{OP}}\right\}^{-1}\Gamma_{\mathrm{OP}}\Delta$$

We call $\mathcal{N}$ the *novation gap* and $\mathcal{N}_u$ the *novation gap towards u*, given by

$$\mathcal{N} = \left(\underline{N}, \overline{N}\right), \qquad \mathcal{N}_u = \mathcal{N} \cap [\underline{k}_u, \overline{k}_u]$$

For a pair i, j that implements an insurance contract, we say i *sells insurance* to j and j *buys insurance* from i if i is a good banker and j is a bad banker.

We denote η_{uv} the mass of bankers in u (bad bankers in u) that buy insurance from a banker in v (good bankers in v), and viceversa. This is an important object because only bankers buying insurance are those that may be restricted regulatory constraints. We also denote π_P the pursuance rate for buying and selling insurance on $P \in \{O, C\}$:

$$\eta_{uv} \coloneqq \mu^{01}_{uv}, \quad \eta_{vu} \coloneqq \mu^{01}_{vu}, \quad \pi_O \coloneqq \pi^{01}_O = \pi(\tau^O), \quad \pi_C \coloneqq \pi^{01}_C = \pi(\tau^C)$$

Lemma 4. *The extensive margins are described by the novation gaps. If banker i buys insurance from j, they novate their contract if and only if $k_i \in \mathcal{N}$.*[24] *This novation gap is empty (this is, $\overline{N} < \underline{N}$) if and only if $\frac{\Gamma_{OP}}{\Gamma_{CR}} > \frac{\delta}{\Delta+\delta}$.*

[22]Recalling net regulatory capital of i, $k_i = \psi^i_R\sigma_i + \left(\psi^i_A(\zeta_i + \mathbb{E}[\alpha]) - m\right)c_{b_i}$, $\overline{k}_u$ and $\underline{k}_u$ the largest and smallest net regulatory capital for a bad banker in u. The purpose of the idiosyncratic component ζ_i is to generate heterogenous preferences within a bank and so we can study intensive margins.

[23]Notice $\tilde{N}_{ij} = \overline{N}$ and $\underset{\sim ij}{N} = \underline{N}$ when $q_i = 0, q_j = 1$.

[24]If $\underline{N} > \overline{N}$, then $\left(\underline{N}, \overline{N}\right) = \emptyset$.

Notice that in the definition of novation gap, $\overline{N}$ only depends on the opacity gap, Γ_{OP}. This is because the novation decision between a pair who is constrained on the OTC but not on the CCP is independent of the CCP-regulation level. In contract, $\underline{N}$ does depend on the regulation gap, Γ_{CR}, As ρ_C increases up from 0 to ρ_O, $\underline{N}$ increases from 0 to ∞, and at some point there is no novation.

Regulatory constraints also affect novation on the intensive margin. To see this, in what follows we characterize the average net positions

Definition. The *average OTC net position towards* u is

$$\begin{aligned}\mathcal{O}_u =&(\overline{k}_u - \underline{k}_u)^{-1}\Big(\delta\left\langle \overline{k}_u - [\![\rho_O\delta]\!]_u\right\rangle + (2\rho_O)^{-1}\left\langle [\![\rho_O\delta]\!]_u^2 - [\![\max\{\overline{N}, \rho_C\delta\}]\!]_u^2\right\rangle \\ &+ (2\rho_O)^{-1}\left\langle [\![\min\{\underline{N}, \rho_C\delta\}]\!]_u^2 - \underline{k}_u^2\right\rangle\Big)\end{aligned}$$

The *average CCP net position towards* u *i*s

$$\mathcal{C}_u =(\overline{k}_u - \underline{k}_u)^{-1}\Big(\delta\langle[\![\overline{N}]\!]_u - [\![\rho_C\delta]\!]_u\rangle + (2\rho_C)^{-1}\langle[\![\rho_C\delta]\!]_u^2 - [\![\underline{N}]\!]_u^2\rangle\Big)$$

where

$$[\![\cdot]\!]_u := \min\{\overline{k}_u, \max\{\underline{k}_u, \cdot\}\}$$

Here $\mathcal{O}_u$ is the average net position of OTC insurance to buyers in u, averaged by the mass of all insurance buying bankers in u. The counterpart for novated contracts is $\mathcal{C}_u$. As a corollary of the expression of $\mathcal{C}_u$ we see that a contract between a good banker and a bad banker $i \in u$ whose net collateral is in a middle range from $[\![\min\{\underline{N}, \rho_C\delta\}]\!]_u$ to $[\![\max\{\overline{N}, \rho_C\delta\}]\!]_u$ are novated. This pins down the novation gap. As $\underline{N} < \rho_C\delta$ and $\overline{N} > \rho_C\delta$ are equivalent, the novation gap is $([\![\underline{N}]\!]_u, [\![\overline{N}]\!]_u)$. The remaining contracts are kept OTC.

Lemma 5. *The intensive margins are described by the average OTC and CCP net positions. Equilibrium exposure scalers are given in closed form as*

$$\begin{aligned}e_{uv}^{**} &= \pi_O\left(\eta_{uv}\mathcal{O}_u - \eta_{vu}\mathcal{O}_v\right)\\ e_{uCv}^{**} &= \pi_C\left(\eta_{uv}\mathcal{C}_u - \eta_{vu}\mathcal{C}_v\right)\end{aligned}$$

Exposure-to-u *scaler is*

$$e_u^{\Rightarrow} = \pi_O\sum_v \overbrace{\langle\eta_{vu}\mathcal{O}_v - \eta_{uv}\mathcal{O}_u\rangle}^{\textit{bilaterally netted}} + \pi_C\ \overbrace{\langle\sum_v(\eta_{vu}\mathcal{C}_v - \eta_{uv}\mathcal{C}_u)\rangle}^{\textit{"multilaterally" netted via CCP}}$$

Figure 10 describes the extensive margins through the novation gap, and intensive margins by the corresponding binding constraint. While $\overline{N}$ increases in Γ_{OP}, $\underline{N}$ decreases in Γ_{OP}. In fact,

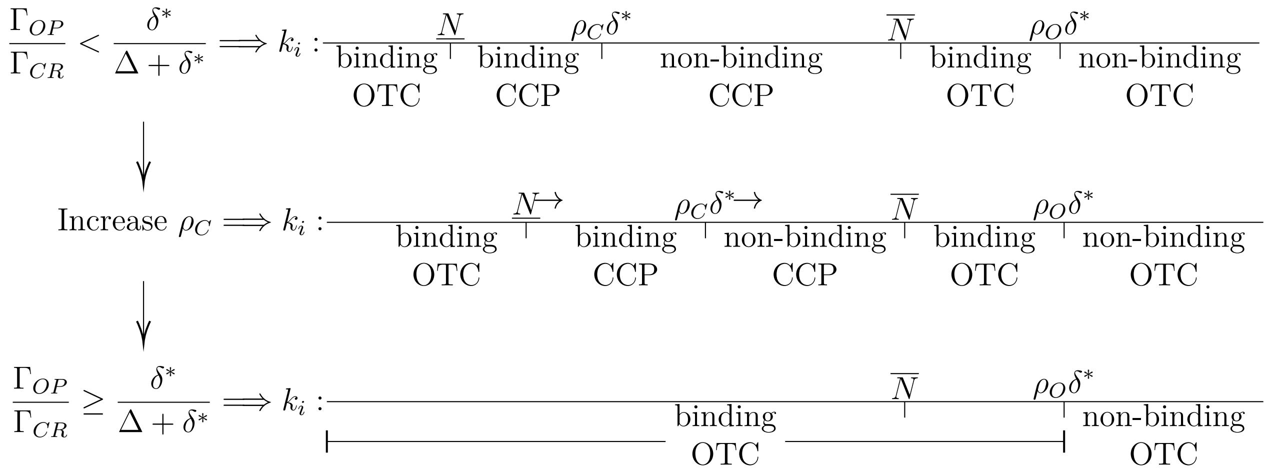

Figure 10: Novation gap as ρ_C increases

the novation thresholds are the sole channel through which opacity impacts extensive margins. A direct implication is that a way to induce more novation is by making CCPs less transparent.

Corollary 1. *The CCP adoption (i.e. the novation gap $\mathcal{N}$), is decreasing in the transparency of the CCP (via increasing the opacity gap Γ_{OP}), down to no adoption (i.e. $\mathcal{N} = \emptyset$) when opacity gap becomes too high relative to regulation gap (i.e. $\Gamma_{OP} \geq \Gamma_{CR}\frac{\delta}{\Delta+\delta}$).*

In words, CCPs introduce transparency in addition of multilateral netting. An opaque clearinghouse who provides netting without transparency would achieve higher rate of adoption. The opacity gap would be 0, and all bankers would adopt the clearinghouse. In a sense, regulation aimed to promote clearing and increase transparency via reduced risk weights for CCP novation makes itself redundant.

Based on these insights, and to put recent implemented regulation in perspective, consider a hypothetical regulator who can choose risk weights as well as the transparency of the CCP, τ^C. Assume his primary objective is full financial stability by eliminating coordination failures, while a secondary objective is to maximize transparency for some political or other non-modeled reasons. The next result shows the minimum opacity gap that is consistent with full CCP adoption that eliminates coordination failures.

Corollary 2. *All exposures of bank u are completely cleared regardless of the matching structure if and only if*

$$\Gamma_{OP} \geq \max\left\{\frac{\Delta + \frac{\overline{k}_u}{\rho_O}}{\Delta+\delta}, \frac{\Delta + \underline{k}_u\frac{1}{\rho_O}}{\Delta + \underline{k}_u\frac{1}{\rho_C}}\right\}$$

In what follows we maintain the opacity gap as given and study the challenges that a regulator faces trying to incentivize clearing just using regulatory requirements. But before discussing the adverse consequences of this type of regulation, we characterize next the conditions under which it is ineffective in inducing clearing, both at intensive and extensive margins.

5.2 The effects of regulation

We first study intensive margins in a systemic way. We focus on a simple and minimal matching structure that does not preclude coordination failures by default.

Definition. Call a matching structure *out-regular* if there is some η and g such that the following hold. For all u, v, $\eta_{uv} \in \{0, \eta\}$ and for all v $|\{u : (\eta_{uv}, \eta_{vu}) = (\eta, 0)\}| = g$.

In words, when $\eta_{uv} = \eta_{vu}$, u and v do not have exposures to each other. When $(\eta_{uv}, \eta_{vu}) = (0, \eta)$, v is exposed to u. Under an out-regular matching structure, bank v can be exposed to any number of banks, but the number of banks exposed to v is g, same for all v. This is, each bank sells insurance to the same number of banks, and while the number of banks that a given bank buys insurance from is not directly limited by out-regularity, the number of insurance sellers and buyers must always add up to the same amount.

Proposition 11. *Suppose that all banks have the same collateral structure (i.e. same mass $\mu_u = \mu$ and caliber $c_u = c$ for all banks). Assume that the matching structure is out-regular with η and g. Then, the coordination failure threshold is $\phi^* = (\mathcal{O}\frac{1}{\mu}\pi_O g\eta + m^*)/c$.*

The simplicity in this case comes form the following observation. Since the total exposures to and from the CCPs are equal, there is at least one bank that does not expose the CCP. This bank's total outgoing OTC exposures, which is fixed by regularity, determines the probability of coordination failures. Hence, the average conditional exposure $\mathcal{O}$ is a sufficient statistic for the efficacy of regulation in reducing coordination failures.

We measure the effects on the average net position of OTC contracts by insurance-buying bankers in u averaged by the mass of OTC insurance buying bankers in u

$$\tilde{\mathcal{O}} = \frac{\mathcal{O}}{\text{OTC prob.}} = \frac{\overline{k} - \underline{k}}{(\overline{k} - \underline{k}) - ([\![\overline{N}]\!] - [\![\underline{N}]\!])}\mathcal{O}$$

The marginal efficacy of increasing ρ_O and decreasing ρ_C is the elasticity of $\tilde{\mathcal{O}}$, defined as

$$\text{Eff}_O = -\frac{\mathrm{d}\tilde{\mathcal{O}}}{\mathrm{d}\rho_O}\frac{\rho_O}{\tilde{\mathcal{O}}} \qquad \text{Eff}_C = \frac{\mathrm{d}\tilde{\mathcal{O}}}{\mathrm{d}\rho_C}\frac{\rho_C}{\tilde{\mathcal{O}}}$$

These elasticities depend on the amount of collateral banks hold. IN what follows we distinguish three ranges: high, medium and low.

Corollary 3. *(High collateral) In addition to Proposition 11 assume high collateral: $\underline{k} > \rho_O\delta$. There is no novation, contracts are unconstrained, and $\mathcal{O} = \delta$. Regulation is ineffective:*

$$\mathit{Eff}_O = 0 \qquad \mathit{Eff}_C = 0$$

Fixing the extensive margins regulation does not bind banks that have high collateral and the novation gap is empty. All contracts are kept OTC and the probability of coordination failures do not depend on regulation on the intensive margin. This result is particularly relevant for the post-Dodd-Frank Era. Further, when there is heterogeneity, high collateral banks are more likely to be the determinants of the probability of coordination failures, since Φ^* is decreasing in c. It is possible that these determinant banks with high enough collateral do not react to regulation.

Corollary 4. *(Medium collateral) In addition to Proposition 11, assume medium collateral, $\rho_O\delta > \overline{k} > \underline{k} > \rho_C\delta$. Then $\mathcal{O} = (4\rho_O Z\psi_A)^{-1}\left\langle \overline{k}^2 - [\![\overline{N}]\!]^2 \right\rangle$.*[25] *For $\underline{k} < \overline{N} < \overline{k}$, all OTC contracts are constrained, but larger contracts are kept OTC whereas smaller contracts are novated. The efficacy of regulation in diminished by medium collateral:*

$$Eff_O = \frac{\overline{k}}{\overline{k} + \overline{N}} \in (\frac{1}{2}, 1) \qquad\qquad Eff_C = 0$$

The inefficacy stems from the fact that high collateral pairs have high value of opacity and are last to switch to CCPs as regulation gets tighter. Formally, when the net regulatory capital is between and $[\![\overline{N}]\!]$ and $\overline{k}$, the value of opacity is positive and the pair keeps the contract OTC, in spite of being constrained by regulation, as reflected in the ρ_O^{-1} term in $\mathcal{O}$. This renders regulation somewhat effective, but not fully given that the average exposure stemming from these contracts is proportional to $\frac{\overline{k}^2 - [\![\overline{N}]\!]^2}{\overline{k} - [\![\overline{N}]\!]} = \overline{k} + [\![\overline{N}]\!]$ which increases in ρ_O.

Corollary 5. *(Low collateral) In addition to Proposition 11, assume low collateral, $\rho_C\delta > \overline{k}$. Then $\mathcal{O} = (4\rho_O Z\psi_A)^{-1}\left\langle [\![\underline{N}]\!]^2 - \underline{k}^2 \right\rangle$.*[26] *For $\underline{k} < \underline{N} < \overline{k}$, all OTC contracts are constrained, and larger contracts are novated whereas smaller contracts are kept OTC. The efficacy of regulation in magnified by low collateral.*

$$Eff_O = \frac{\underline{N}}{\underline{N} + \underline{k}} \frac{1 - \Gamma_{CR}}{\Gamma_{CR} - \Gamma_{OP}} + 1 > \frac{3}{2} \qquad\qquad Eff_C = \frac{\underline{N}}{\underline{N} + \underline{k}} \frac{1 - \Gamma_{OP}}{\Gamma_{CR} - \Gamma_{OP}} > \frac{1}{2}$$

As in the case of medium collateral, contracts are constrained by OTC regulation reflected in the ρ_O^{-1} term in $\mathcal{O}$. This makes regulation effective. Additionally, contracts with larger net positions move to novation first and the average exposure stemming from contracts that are kept OTC is $\frac{\underline{N}^2 - \underline{k}^2}{\underline{N} - \underline{k}} = \underline{N} + \underline{k}$ which decreases in ρ_O. This adds efficacy to regulation.

In summary, the effects of regulation depends critically on the amount of collateral. Regulation has a larger impact when buyers of insurance are relative scarce on collateral. When there is heterogeneity, regulation can potentially be less effective in segments of the network that

[25] For $\overline{N} > \overline{k}$, all contracts are novated. There is no coordination failure and efficacy is not well-defined. For $\overline{N} < \underline{k}$, there is no novation and OTC capital constraints bind all contracts. $\text{Eff}_O = 1$, $\text{Eff}_C = 0$.

[26] For $\underline{N} < \underline{k}$, all contracts are novated. There is no coordination failure and efficacy is not well-defined. For $\underline{N} > \overline{k}$, there is no novation and OTC capital constraints bind all contracts. $\text{Eff}_O = 1$, $\text{Eff}_C = 0$.

is highly collateralized. If there is a positive correlation between high collateral and interconnectedness, it is possible that parts of the network with cycles, which are the more important targets for regulation, are the least likely to react to regulation. In fact, our data also shows that post-regulation, OTC exposures in the core of the network is backed by high collateral and the core is highly interconnected with cyclic exposures.

6 Conclusion

Recent regulations that tax the use of OTC insurance contracts and subsidize those that use CCPs have had mixed success and much discussion. In order to make progress on this discussion, it is critical to understand how financial intermediaries connect with each other and how large are those connections. We have proposed a model to understand the effects of regulation and capture asymmetric reactions across heterogeneous banks. While OTC markets maintain opacity about insurance needs and then allow for insurance without affecting funding costs, they are more exposed to coordination failures that may drive the whole banking system to collapse. While banks internalize the benefits of OTC contracts, they do not internalize the coordination costs. This justifies the regulation that tries to incentivize the use of CCPs. However, when the gains of opacity are larger than the regulatory costs, which is usually the case when a bank has frequent needs for insurance, regulations are not very effective. These are, however, the banks that are more likely to trigger coordination failures.

Modeling coordination failures and what they depend on requires a setting with endogenous formation of exposures. A contribution of this paper is proposing a tractable model in which this endogenous formation, both at extensive and intensive margins, can be captured parsimoniously.

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A Proofs

Proof. **(Theorem 1)** In all contagion outcomes $\tilde{E}^{\leftarrow}_{u,\alpha} \leq E^{\leftarrow}_{u,\alpha}$. Then if $E^{\leftarrow}_{u,\alpha} = 0$, $\tilde{E}^{\leftarrow}_{u,\alpha} = 0$. Then in condition (1) holds or does not hold irrespective of the contagion outcome. This is, u either always defaults or never defaults in any contagion outcome. So there can not be a weak coordination failure.

This means $E^{\leftarrow}_{u,\alpha} > 0$ for all u if there is a weak coordination failure. Then start with bank u_0 and construct a sequence as $E'_{u_t u_{t+1}} > 0$. As there a finitely many banks, there is some $y < y'$

such that $u_y = u_{y'}$. Then there is a directed cycle of exposures from u_y to $u_{y'-1}$. If $y = 0$, then u_0 is on a cycle. Otherwise u_0 is indirectly exposued to the cycle through the path u_0 to u_y. □

Proof. **(Proposition 1)** First requirement for a coordination failure is that all banks defaulting and paying each other 0 is a contagion outcome. This is true if and only if for all banks, the bank defaults if it receives 0 payments from other institutions, and the bank can pay 0 to other institutions whenever it receives zero from other institutions and defaults. The conditions are then given by

$$0 > \alpha A_u + R_u - L_u - E_u^{\rightarrow}$$
$$0 \geq \lambda_A \alpha A_u + \lambda_R R_u - L_u$$

These are generically equivalent to the left hand sides of inequalities (2).

A banker in $i \in u$ when can not promise more than rs' when signing a contract with another banker. Accordingly, $E_u^{\rightarrow} \leq R_u s'$. Under $\lambda_R < s$, $E_u^{\rightarrow} \leq R_u s' \leq R_u(1 - \lambda_R)$. Then $0 > \alpha A_u + R_u - E_u^{\rightarrow} - L_u$ implies $0 > \lambda_A \alpha A_u + \lambda_R R_u - L_u$.

The second requirement is that no banks default is also a contagion outcome. This is given by $Q_{u,\alpha} \geq 0$ which is equivalent to the left hand side of inequality (2).

The remaining parts are straightforward. □

Proof. **(Proposition 5)** Proposition 2 and the independence of w_i and w_j yields the result. □

Proof. **(Proposition 6)** On the path of play there are no withdrawals. All bankers who have been financed remain financed. The collateral is liquidated for bankers who fail to secure financing and the credit is returned to the creditor. For $i \in u$, the probability of financing is $\kappa_i(0) = \pi_i(0)\Omega_u\varpi^{-1}$. Then $A_i = \int a_i \mathrm{d}i = \int \kappa_i(0)c_u \mathrm{d}i = c_u\Omega_u\omega^*\varpi^{-1}\int \pi_i(0)\mathrm{d}i = \Omega_u a_u^*$. Positive project returns are contingent on financing ($\kappa_i(0)$) and project success ($\pi_i(0)$): $R_u = \int_u \kappa_i(0)\pi_i(0)r\mathrm{d}i = r\Omega_u\omega^*\varpi^{-1}\int(\pi_i(0))^2\mathrm{d}i = \Omega_u r_u^*$. Similarly, $L_u = sr_u^* + m^*$.

We know that the probability mass of $i \in u$ and $j \in v$ having qualities q and q', and getting matched is $\gamma_{iq}\gamma_{jq'}\tilde{\mu}_{uv}^{qq'}$. The probability of insurance is $\kappa_i^{\tau_{ij}}\kappa_j^{\tau_{ji}}$. The net expected payment flow from j to i is $D_{ij}^{qq'}$. Then

$$\begin{aligned}
E_{uv} - E_{vu} &= \sum_{q,q'} \int_{i\in u} \int_{j\in v} \kappa_i^{\tau_{ij}} \kappa_j^{\tau_{ji}} D_{ij}^{qq'} \gamma_{iq}\gamma_{jq'} \tilde{\mu}_{uv}^{qq'} \mathtt{d}j\mathtt{d}i \\
&- \sum_{q',q} \int_{j\in v} \int_{i\in u} \kappa_i^{\tau_{ij}} \kappa_j^{\tau_{ji}} D_{ji}^{q'q} \gamma_{iq}\gamma_{jq'} \tilde{\mu}_{uv}^{qq'} \mathrm{d}i\mathrm{d}j \\
&= \sum_{q,q'} \int_{i\in u} \int_{j\in v} \kappa_i^{\tau_{ij}} \kappa_j^{\tau_{ji}} \left(D_{ij}^{qq'} - D_{ji}^{q'q} \right) \gamma_{iq}\gamma_{jq'} \tilde{\mu}_{uv}^{qq'} \mathtt{d}j\mathtt{d}i \\
&= \Omega_u \Omega_v \overline{\omega}^{-2} \omega^{*2} \sum_{q,q'} \int_{i\in u} \int_{j\in v} \pi_{ij}^{qq'} \left(D_{ij}^{qq'} - D_{ji}^{q'q} \right) \gamma_{iq}\gamma_{jq'} \tilde{\mu}_{uv}^{qq'} \mathtt{d}j\mathtt{d}i \\
&= \Omega_u \Omega_v e_{uv}^*
\end{aligned}$$

□

Proof. **(Proposition 7)** Fix a platform and drop the platform index. Wlog take $\beta_{ji} \leq \beta_{ij}$. First ignore IC. The capital-unconstrained optimal is $(D_{ij}, D_{ji}) = (\beta_{ij}, \beta_{ji})$ by $\theta(\sigma_{ji} - \xi) > \sigma_j - \sigma_i$. As $k_j/\rho_{ji} \geq 0 \geq \beta_{ji} - \beta_{ij}$, j's capital constraint does not bind. If $k_i/\rho_{ij} \geq \beta_{ij} - \beta_{ji}$, then the solution is (β_{ij}, β_{ji}) regardless of ξ.

Consider the case of $\beta_{ji} \leq \beta_{ij}$ and $k_i/\rho_{ij} < \beta_{ij} - \beta_{ji}$. Total expected contracting gain is $\theta(\sigma_{ij}\min\{\beta, d_{ij}\} + \sigma_{ji}\min\{\beta, d_{ji}\} - \xi d_{ij} - \xi d_{ji})$. This is decreasing in (d_{ij}, d_{ji}) on $(d_{ij}, d_{ji}) \geq (\beta, \beta)$ and increasing on $(d_{ij}, d_{ji}) \leq (\beta, \beta)$. As $k_i/\rho_{ij} < \beta_{ij} - \beta_{ji}$, the region $d_{ij} \geq \beta$,$d_{ji} \leq \beta$ violates i's capital constraint. So the solution is on $d_{ij} \leq \beta$, $d_{ji} \geq \beta$. In this region, Total expected contracting gain is $\theta(\sigma_{ij}d_{ij} + \sigma_{ji}\beta - \xi d_{ij} - \xi d_{ji})$, which is increasing in $d_{ij} \in [0, \beta]$ and decreasing in $d_{ji} \in [0, \beta]$. The capital constraint of i is $\sigma_{ij}d_{ij} \leq k_i/\rho_{ij} + \sigma_{ji}d_{ji}$. So the solution is on the border of the constraint. The slope of the indifference curve $\frac{\sigma_{ij}-\xi}{\xi}$ is larger than the slope of the border of the constraint $\frac{\sigma_{ij}}{\sigma_{ji}}$ if and only if

$$\frac{\sigma_{ij}}{\sigma_{ji}} < \frac{\sigma_{ij} - \xi}{\xi} \iff \xi(\sigma_{ij} + \sigma_{ji}) < \sigma_{ij}\sigma_{ji} \iff \xi < \xi_{ij}^*$$

Now consider the case of $\xi < \xi_{ij}^*$. In this case, the solution is given maximizing d_{ij}, so $d_{ij} = \beta$ and $(\sigma_{ij}\beta - k_i/\rho_{ij})/\sigma_{ji} = d_{ji}$. By the satisfied capital constraint of j at (β, β) and the violated capital constraint of i at (β, β) we have

$$\max\left\{\beta, \frac{\beta_{ji} - k_j/\rho_{ji}}{\sigma_{ij}}\right\} = \beta = d_{ij}, \quad \frac{\beta_{ij} - k_i/\rho_{ij}}{\sigma_{ji}} = \max\left\{\beta, \frac{\beta_{ij} - k_i/\rho_{ij}}{\sigma_{ji}}\right\} = d_{ji}$$

For the case of $\xi > \xi^*$, the solution is given by minimizing d_{ji}, so $d_{ji} = \beta$ and $d_{ij} = \frac{\sigma_{ji}\beta + k_i/\rho_{ij}}{\sigma_{ij}}$. By the violated capital constraint of i at (β, β) and the satisfied capital constraint of j at (β, β)

this can be written as

$$\beta > \min\left\{\beta, \frac{\beta_{ji} + k_i/\rho_{ij}}{\sigma_{ij}}\right\} = \frac{\beta_{ji} + k_i/\rho_{ij}}{\sigma_{ij}} = d_{ij}, \quad \min\left\{\beta, \frac{\beta_{ij} + k_j/\rho_{ji}}{\sigma_{ji}}\right\} = \beta = d_{ji}$$

Last step is to check the IC. As $d_{ij} \leq \beta$ and $d_{ji} \geq \beta$, the IC constraints are

$$0 \leq^? \sigma_{ij}(d_{ij} + \theta d_{ij}) - (\sigma_{ji} + \xi\theta)d_{ji} = k_i/\rho_{ij} + \theta(k_i/\rho_{ij} + (\sigma_{ji} - \xi)d_{ji}) > 0$$
$$0 \leq^? \sigma_{ji}(d_{ji} + \theta\beta) - (\sigma_{ij} + \xi\theta)d_{ij} \geq \sigma_{ji}(\beta + \theta\beta) - (\sigma_{ij} + \xi\theta)\beta = \beta(\theta(\sigma_{ji} - \xi) - (\sigma_j - \sigma_i)) > 0$$

So both are satisfied. □

Proof. **(Proposition 9)** By Proposition 7 and the definition of Θ in Lemma 3. The rest is straightforward algebra that proves $k_i < \tilde{N}_{ij} \iff \Theta < 0$ under medium collateral, and $k_i > \underset{\sim}{N}_{ij} \iff \Theta < 0$ under low collateral. □

Lemma 6. *(Auxiliary) Suppose all bankers are ex-ante identical within a bank. This means:* $\sigma_{i0} \equiv \sigma_{u0}$, $\sigma_{i1} \equiv \sigma_{u0}$, $\gamma_i \equiv \gamma_u$, $\tau^O_{ij} \equiv \tau^O_{uv}$, $\tau^C_{ij} \equiv \tau^C_{uv}$, $RW^i_R \equiv RW^u_R$, *and* $RW^{ij}_{O/C} = RW^{uv}_{O/C}$ *for all* $i \in u, j \in v$.

Denote $\mathbb{I}^{qq'}_{uv} = \mathbb{I}[\sigma_{vq'} - \sigma_{uq} > 0]$ *and define*

$$\mathcal{O}^{qq'}_{uv} = \frac{1}{2Z\psi^u_A c_u}\left(\delta^{qq'}_{uv}\left(\overline{k}_u - \left[\!\left[\rho^{uv}_O \delta^{qq'}_{uv}\right]\!\right]_u\right) + (2\rho^{uv}_O)^{-1}\left(\left(\left[\!\left[\rho^{uv}_O \delta^{qq'}_{uv}\right]\!\right]^2 - \Lambda_u(\max\{\overline{N}_{uv}, \rho^{uv}_C \delta^{qq'}_{uv}\})^2\right)^+ + \left(\Lambda_u(\min\left\{\underline{N}_{uv}, \rho^{uv}_C \delta^{qq'}_{uv}\right\})^2 - \underline{k}^2_u\right)^+\right)\right)$$

$$\mathcal{C}^{qq'}_{uv} = \frac{1}{2Z\psi^u_A c_u}\left(\delta^{qq'}_{uv}\left(\Lambda_u(\min\left\{\overline{N}_{uv}, \rho^{uv}_O \delta^{qq'}_{uv}\right\}) - \Lambda_u(\rho^{uv}_C \delta^{qq'}_{uv})\right)^+ \mathcal{C}^{qq'}_{uv} + (2\rho^{uv}_C)^{-1}\left(\Lambda_u(\rho^{uv}_C \delta^{qq'}_{uv})^2 - \Lambda_u(\underline{N}_{uv})^2\right)^+\right)$$

Then exposure scalers are given by

$$e^{**}_{uv} = \varpi^{-2}\sum_{qq'} \mu^{qq'}_{uv} \pi^{qq'}_{uvO}\left(\mathbb{I}^{qq'}_{uv}\mathcal{O}^{qq'}_{uv} - \mathbb{I}^{q'q}_{vu}\mathcal{O}^{q'q}_{vu}\right)$$

$$e^{**}_{uCv} = \varpi^{-2}\sum_{qq'} \mu^{qq'}_{uv} \pi^{qq'}_{uvO}\left(\mathbb{I}^{qq'}_{uv}\mathcal{C}^{qq'}_{uv} - \mathbb{I}^{q'q}_{vu}\mathcal{C}^{q'q}_{vu}\right)$$

Proof. **(Lemma 6)** Suppose $\beta_{ji} \leq \beta_{ij}$ (note $0 \leq \delta_{ij} := \beta_{ij} - \beta_{ji} = \beta(\sigma_j - \sigma_i)$). The expected payments in the contract and novation choice between i and j conditional on pursuability is summarized as:

$(D_{Pij}, D_{Pji}, D_{Pij}-D_{Pji})$	$\beta_{ij}-\frac{k_i}{\rho_C^{ij}}<\beta_{ij}-\frac{k_i}{\rho_O^{ij}}<\beta_{ji}$	$\beta_{ij}-\frac{k_i}{\rho_C^{ij}}<\beta_{ji}<\beta_{ij}-\frac{k_i}{\rho_O^{ij}}$	$\beta_{ji}<\beta_{ij}-\frac{k_i}{\rho_C^{ij}}<\beta_{ij}-\frac{k_i}{\rho_O^{ij}}$
	$(\beta_{ij},\beta_{ji},\delta_{ij})_O$	NA	NA
$k_i<\tilde{N}_{ij}\ (\Theta_{ij}<0)$	NA	$(\beta_{ij},\beta_{ji},\delta_{ij})_C$	NA
$k_i>\tilde{N}_{ij}\ (\Theta_{ij}>0)$	NA	$\left(\beta_{ij},\beta_{ji}-\frac{k_i}{\rho_O^{ij}},\frac{k_i}{\rho_O^{ij}}\right)_O$	NA
$k_i>\underset{\sim}{N}_{ij}\ (\Theta_{ij}<0)$	NA	NA	$\left(\beta_{ij},\beta_{ji}-\frac{k_i}{\rho_C^{ij}},\frac{k_i}{\rho_C^{ij}}\right)_C$
$k_i<\underset{\sim}{N}_{ij}\ (\Theta_{ij}>0)$	NA	NA	$\left(\beta_{ij},\beta_{ji}-\frac{k_i}{\rho_O^{ij}},\frac{k_i}{\rho_O^{ij}}\right)_O$

Recall definitions of $e_{uv}^{**}, e_{uC}^{**}, e_{Cu}^{**}$. Note $x=x^++x^-=x^+-(-x)^+$. Then

$$\begin{aligned}
e_{uv}^{**}=&\overline{\omega}^{-2}\sum_{q_iq_j}\int_{i\in u}\int_{j\in v}\pi_{ijO}^{q_iq_j}(D_{Oij}^{q_iq_j}-D_{Oji}^{q_jq_i})\{\{\Theta_{ij}^{q_iq_j}>0\}\}\gamma_{iq_i}\gamma_{jq_j}\tilde{\mu}_{uv}^{q_iq_j}\mathtt{d}j\mathtt{d}i\\
=&\overline{\omega}^{-2}\sum_{q_iq_j}\int_{i\in u}\int_{j\in v}\pi_{ijO}^{q_iq_j}(D_{Oij}^{q_iq_j}-D_{Oji}^{q_jq_i})^+\{\{\Theta_{ij}^{q_iq_j}>0\}\}\gamma_{iq_i}\gamma_{jq_j}\tilde{\mu}_{uv}^{q_iq_j}\mathtt{d}j\mathtt{d}i\\
&-\overline{\omega}^{-2}\sum_{q_iq_j}\int_{i\in u}\int_{j\in v}\pi_{ijO}^{q_iq_j}(D_{Oji}^{q_jq_i}-D_{Oij}^{q_iq_j})^+\{\{\Theta_{ij}^{q_iq_j}>0\}\}\gamma_{iq_i}\gamma_{jq_j}\tilde{\mu}_{uv}^{q_iq_j}\mathtt{d}j\mathtt{d}i
\end{aligned}$$

and

$$\begin{aligned}
e_{Cv}^{**}=&\overline{\omega}^{-2}\sum_{u}\sum_{q_iq_j}\int_{i\in u}\int_{j\in v}\pi_{ijC}^{q_iq_j}(D_{Cij}^{q_iq_j}-D_{Cji}^{q_jq_i})\{\{\Theta_{ij}^{q_iq_j}<0\}\}\gamma_{iq_i}\gamma_{jq_j}\tilde{\mu}_{uv}^{q_iq_j}\mathtt{d}j\mathtt{d}i\\
=&\overline{\omega}^{-2}\sum_{u}\sum_{q_iq_j}\int_{i\in u}\int_{j\in v}\pi_{ijC}^{q_iq_j}(D_{Cij}^{q_iq_j}-D_{Cji}^{q_jq_i})^+\{\{\Theta_{ij}^{q_iq_j}<0\}\}\gamma_{iq_i}\gamma_{jq_j}\tilde{\mu}_{uv}^{qq_iq_j}\mathtt{d}j\mathtt{d}i\\
&-\overline{\omega}^{-2}\sum_{u}\sum_{q_iq_j}\int_{i\in u}\int_{j\in v}\pi_{ijC}^{q_iq_j}(D_{Cji}^{q_jq_i}-D_{Cij}^{q_iq_j})^+\{\{\Theta_{ij}^{q_iq_j}<0\}\}\gamma_{iq_i}\gamma_{jq_j}\tilde{\mu}_{uv}^{qq_iq_j}\mathtt{d}j\mathtt{d}i
\end{aligned}$$

Recall $\tilde{\mu}_{uv}^{qq'}=\frac{\mu_{uv}^{qq'}}{\mu_u\gamma_{uq}\mu_v\gamma_{vq'}}$. As bankers in a bank are ex-ante identical

$$\begin{aligned}
&\int_{i\in u}\int_{j\in v}\pi_{ijP}^{q_iq_j}\left(D_{Pij}^{q_iq_j}-D_{Pji}^{q_jq_i}\right)^+\{\{\pm\Theta_{ij}^{q_iq_j}<0\}\}\gamma_{iq}\gamma_{jq'}\tilde{\mu}_{uv}^{q_iq_j}\mathtt{d}j\mathtt{d}i\\
=&\frac{\mu_{uv}^{q_iq_j}\pi_{uvP}^{q_iq_j}}{\mu_u\mu_v}\int_{i\in u}\int_{j\in v}\left(D_{Pij}^{q_iq_j}-D_{Pji}^{q_jq_i}\right)^+\{\{\pm\Theta_{ij}^{q_iq_j}<0\}\}\mathtt{d}j\mathtt{d}i\\
=&\frac{\mu_{uv}^{q_iq_j}\pi_{uvP}^{q_iq_j}}{\mu_u\mu_v}\int_{i\in u}\int_{j\in v}\left(D_{Pij}^{q_iq_j}-D_{Pji}^{q_jq_i}\right)\mathbb{I}[\sigma_i<\sigma_j]\{\{\pm\Theta_{ij}^{q_iq_j}<0\}\}\mathtt{d}j\mathtt{d}i
\end{aligned}$$

Then $\mathbb{I}[\sigma_i<\sigma_j]$ determines the direction of the contribution to the exposure between u and v. $\{\{\pm\Theta_{ij}^{q_iq_j}<0\}\}$ determines the platform for i and j. Denote $\mathbb{I}_{ij}=\mathbb{I}[\sigma_j-\sigma_i>0]$.

Regarding $\mathbb{I}_{ij}$, for $i\in u, j\in v$, δ_{ij} has four cases depending one the quality of the bankers: $\sigma_{v0}-\sigma_{u1}<(\sigma_{v1}-\sigma_{u1}\&\sigma_{v0}-\sigma_{u0})<\sigma_{v1}-\sigma_{u0}$. In terms of which one of these are positive or negative, there are 6 cases to consider. Regarding $\left(D_{Pij}^{q_iq_j}-D_{Pji}^{q_jq_i}\right)^+$, the relevant cases are when $\mathbb{I}_{ij}=\mathbb{I}_{uv}^{q_iq_j}>0$.

In all of these cases, for some q, q', conditional on $q_i = q, q_j = q'$ (i.e. conditional on a specific $\delta_{ij} = \delta_{uv}^{qq'}$) such that $\mathbb{I}_{uv}^{qq'} = 1$, notice that $(D_{Pij} - D_{Pji})\,\mathbb{I}_{ij}\{\{\pm\Theta_{ij}^{qq'} < 0\}\}$ does not depend on the identity of j. $D_{Pij} - D_{Pji} \in \{\delta_{uv}^{qq'}, \frac{k_i}{\rho_O^{uv}}, \frac{k_i}{\rho_C^{uv}}\}$. $\{\{\pm\Theta_{ij}^{qq'} < 0\}\}$ captures the platform choice through $\Theta_{ij} = \Theta_{iv}^{qq'}$, which depends on k_i but not k_j. So conditional on qualities $q_i = q, q_j = q'$,

$$
\begin{aligned}
&\int_{i\in u}\int_{j\in v}\left(D_{Pij}^{qq'} - D_{Pji}^{qq'}\right)\mathbb{I}[\sigma_i < \sigma_j]\{\{\pm\Theta_{ij}^{qq'} < 0\}\}\mathtt{d}j\mathtt{d}i \\
=&\int_{i\in u}\int_{j\in v}\{\delta_{uv}^{qq'}, \frac{k_i}{\rho_O^{uv}}, \frac{k_i}{\rho_C^{uv}}\}\mathbb{I}_{uv}^{qq'}\{\{\pm\Theta_{iv}^{qq'} < 0\}\}\mathtt{d}j\mathtt{d}i \\
=&\mu_v\int_{i\in u}\{\delta_{uv}^{qq'}, \frac{k_i}{\rho_O^{uv}}, \frac{k_i}{\rho_C^{uv}}\}\mathbb{I}_{uv}^{qq'}\{\{\pm\Theta_{iv}^{qq'} < 0\}\}\mathtt{d}i
\end{aligned}
$$

Recalling $k_i = \psi_R^i\sigma_i + (\psi_A^i(\zeta_i + \mathbb{E}[\alpha]) - m)\,c_{b_i}$, we have $k_i = \psi_R^u\sigma_{uq} + (\psi_A^u(\zeta_i + \mathbb{E}[\alpha]) - m)\,c_u$, we have

$$
\begin{aligned}
&\int_{i\in u}\int_{j\in v}\left(D_{Pij}^{qq'} - D_{Pji}^{qq'}\right)\mathbb{I}[\sigma_i \le \sigma_j]\{\{\pm\Theta_{iv}^{qq'} < 0\}\}\mathtt{d}j\mathtt{d}i \\
=&\frac{\mu_u\mu_v}{2Z\psi_A^u c_u}\int_{\underline{k}_u}^{\overline{k}_u}\{\delta_{uv}^{qq'}, \frac{k_i}{\rho_O^{uv}}, \frac{k_i}{\rho_C^{uv}}\}\mathbb{I}_{uv}^{qq'}\{\{\pm\Theta_{iv}^{qq'} < 0\}\}dk_i
\end{aligned}
$$

Denote $\Lambda_i(\cdot) = \min\{\overline{k}_i, \max\{\underline{k}_i, \cdot\}\}$. Since bankers are ex-ante identical within a bank, we use $\Lambda_u = \min\{\overline{k}_u, \max\{\underline{k}_u, \cdot\}\}$. Then conditional on $(q_i = q, q_j = q')$, $\int\int\left(D_{Pij}^{q_iq_j} - D_{Pji}^{q_jq_i}\right)^+\zeta_{ij}\mathtt{d}j$ is 0 if $\mathbb{I}_{uv}^{qq'} = 0$, and given by the following if $\mathbb{I}_{uv}^{qq'} = 1$:

$(\frac{\mu_u\mu_v}{2Z\psi_A^u c_u})^{-1}\int\int\left(D_{Pij}^{q_iq_j} - D_{Pji}^{q_jq_i}\right)^+\zeta_{ij}\mathtt{d}j\mathtt{d}i$	$\rho_O^{uv}\delta_{uv}^{qq'} < k_i$	$\rho_C^{uv}\delta_{uv}^{qq'} < k_i < \rho_O^{uv}\delta_{uv}^{qq'}$	$k_i < \rho_C^{uv}\delta_{uv}^{qq'}$
	$O: \int_{\Lambda_u(\rho_O^{uv}\delta_{uv}^{qq'})}^{\overline{k}_u}\delta_{uv}^{qq'}dk_i$	NA	NA
$k_i > \overline{N}_{uv}$ $(\Theta_{iv} > 0)$	NA	$O: \int_{\Lambda_u(\max\{\overline{N}_{uv}, \rho_C^{uv}\delta_{uv}^{qq'}\})}^{\Lambda_u(\rho_O^{uv}\delta_{uv}^{qq'})}\frac{k_i}{\rho_O^{ij}}dk_i$	NA
$k_i < \overline{N}_{uv}$ $(\Theta_{iv} < 0)$	NA	$C: \int_{\Lambda_u(\rho_C^{uv}\delta_{uv}^{qq'})}^{\Lambda_u(\min\left\{\overline{N}_{uv}, \rho_O^{uv}\delta_{uv}^{qq'}\right\})}\delta_{ij}dk_i$	NA
$k_i > \underline{N}_{uv}$ $(\Theta_{iv} < 0)$	NA	NA	$C: \int_{\Lambda_u(\underline{N}_{uv})}^{\Lambda_u(\rho_C^{uv}\delta_{uv}^{qq'}\})}\frac{k_i}{\rho_C^{ij}}dk_i$
$k_i < \underline{N}_{uv}$ $(\Theta_{iv} > 0)$	NA	NA	$O: \int_{\underline{k}_u}^{\Lambda_u\min\left\{\underline{N}_{uv}, \rho_C^{uv}\delta_{uv}^{qq'}\right\}}\frac{k_i}{\rho_O^{ij}}dk_i$

Then

$$\sum_{q_iq_j}\int_{i\in u}\int_{j\in v}\pi^{q_iq_j}_{ijO}(D^{q_iq_j}_{Oij}-D^{q_jq_i}_{Oji})^+\zeta\{\Theta^{q_iq_j}_{ij}\}\gamma_{iq_i}\gamma_{jq_j}\tilde{\mu}^{q_iq_j}_{uv}\mathtt{d}j\mathtt{d}i$$
$$=\sum_{qq'}\mathbb{I}^{qq'}_{uv}\frac{\mu^{qq'}_{uv}\pi^{qq'}_{uvO}}{\mu_u\mu_v}\frac{\mu_u\mu_v}{2Z\psi^u_A c_u}(((\int_{\Lambda_u(\rho^{uv}_O\delta^{qq'}_{uv})}^{\overline{k}_u}\delta^{qq'}_{uv}dk_i+\int_{\Lambda_u(\max\{\overline{N}_{uv},\rho^{uv}_C\delta^{qq'}_{uv}\})}^{\Lambda_u(\rho^{uv}_O\delta^{qq'}_{uv})}\frac{k_i}{\rho^{uv}_O}dk_i+\int_{\underline{k}_u}^{\Lambda_u\min\left\{\underline{N}_{uv},\rho^{uv}_C\delta^{qq'}_{uv}\right\}}\frac{k_i}{\rho^{uv}_O}dk_i)))$$
$$=\sum_{qq'}\mathbb{I}^{qq'}_{uv}\mu^{qq'}_{uv}\pi^{qq'}_{uvO}\frac{1}{2Z\psi^u_A c_u}(((\delta^{qq'}_{uv}\left(\overline{k}_u-\Lambda_u(\rho^{uv}_O\delta^{qq'}_{uv})\right)$$
$$+(2\rho^{uv}_O)^{-1}\left(\left(\Lambda_u(\rho^{uv}_O\delta^{qq'}_{uv})^2-\Lambda_u(\max\{\overline{N}_{uv},\rho^{uv}_C\delta^{qq'}_{uv}\})^2\right)^+ +\left(\Lambda_u(\min\left\{\underline{N}_{uv},\rho^{uv}_C\delta^{qq'}_{uv}\right\})^2-\underline{k}^2_u\right)^+\right))))$$
$$=:\sum_{qq'}\mathbb{I}^{qq'}_{uv}\mu^{qq'}_{uv}\pi^{qq'}_{uvO}\mathcal{O}^{qq'}_{uv}$$

where

$$\mathcal{O}^{qq'}_{uv}=\frac{1}{2Z\psi^u_A c_u}\left(\delta^{qq'}_{uv}\left(\overline{k}_u-\Lambda_u(\rho^{uv}_O\delta^{qq'}_{uv})\right)+(2\rho^{uv}_O)^{-1}\left(\left(\Lambda_u(\rho^{uv}_O\delta^{qq'}_{uv})^2-\Lambda_u(\max\{\overline{N}_{uv},\rho^{uv}_C\delta^{qq'}_{uv}\})^2\right)^+ +\left(\Lambda_u(\min\left\{\underline{N}_{uv},\rho^{uv}_C\delta^{qq'}_{uv}\right\})^2-\underline{k}^2_u\right)^+\right)\right)$$

These are exposures of u to v from contracts in which v owes u in expectation. Subtract from this the the contracts in which u owes v in expectation to find e^{**}_{uv}. Then

$$e^{**}_{uv}=\varpi^{-2}\sum_{qq'}\mu^{qq'}_{uv}\pi^{qq'}_{uvO}\left(\mathbb{I}^{qq'}_{uv}\mathcal{O}^{qq'}_{uv}-\mathbb{I}^{q'q}_{vu}\mathcal{O}^{q'q}_{vu}\right)$$

As for the CCP exposures,

$$\sum_{q_iq_j}\int_{i\in u}\int_{j\in v}\pi^{q_iq_j}_{ijC}(D^{q_iq_j}_{Cij}-D^{q_jq_i}_{Cji})^+\zeta\{-\Theta^{q_iq_j}_{ij}\}\gamma_{iq_i}\gamma_{jq_j}\tilde{\mu}^{q_iq_j}_{uv}\mathtt{d}j\mathtt{d}i$$
$$=\sum_{qq'}\mathbb{I}^{qq'}_{uv}\frac{\mu^{qq'}_{uv}\pi^{qq'}_{uvC}}{\mu_u\mu_v}\frac{\mu_u\mu_v}{2Z\psi^u_A c_u}\left(\int_{\Lambda_u(\rho^{uv}_C\delta^{qq'}_{uv})}^{\Lambda_u(\min\left\{\overline{N}_{uv},\rho^{uv}_O\delta^{qq'}_{uv}\right\})}\delta^{qq'}_{uv}dk_i+\int_{\Lambda_u(\underline{N}_{uv})}^{\Lambda_u(\rho^{uv}_C\delta^{qq'}_{uv})}\frac{k_i}{\rho^{uv}_C}dk_i\right)$$
$$=\sum_{qq'}\mathbb{I}^{qq'}_{uv}\mu^{qq'}_{uv}\pi^{qq'}_{uvC}\frac{1}{2Z\psi^u_A c_u}\left(\delta^{qq'}_{uv}\left(\Lambda_u(\min\left\{\overline{N}_{uv},\rho^{uv}_O\delta^{qq'}_{uv}\right\})-\Lambda_u(\rho^{uv}_C\delta^{qq'}_{uv})\right)^+\right.$$
$$=\left.+(2\rho^{uv}_C)^{-1}\left(\Lambda_u(\rho^{uv}_C\delta^{qq'}_{uv})^2-\Lambda_u(\underline{N}_{uv})^2\right)^+\right)$$
$$=:\sum_{qq'}\mathbb{I}^{qq'}_{uv}\mu^{qq'}_{uv}\pi^{qq'}_{uvO}\mathcal{C}^{qq'}_{uv}=\sum_{qq'}\mathbb{I}^{qq'}_{uv}\mu^{qq'}_{uv}\pi^{qq'}_{uvO}\mathcal{C}^{qq'}_{uv}$$

where

$$\mathcal{C}^{qq'}_{uv}=\frac{1}{2Z\psi^u_A c_u}\left(\delta^{qq'}_{uv}\left(\Lambda_u(\min\left\{\overline{N}_{uv},\rho^{uv}_O\delta^{qq'}_{uv}\right\})-\Lambda_u(\rho^{uv}_C\delta^{qq'}_{uv})\right)^+\mathcal{C}^{qq'}_{uv}+(2\rho^{uv}_C)^{-1}\left(\Lambda_u(\rho^{uv}_C\delta^{qq'}_{uv})^2-\Lambda_u(\underline{N}_{uv})^2\right)^+\right)$$

This implies

$$e_{uCv}^{**} = \overline{\omega}^{-2} \sum_{qq'} \mu_{uv}^{qq'} \pi_{uvO}^{qq'} \left(\mathbb{I}_{uv}^{qq'} \mathcal{C}_{uv}^{qq'} - \mathbb{I}_{vu}^{q'q} \mathcal{C}_{vu}^{q'q} \right)$$

$\square$

Lemma 7. *(Auxiliary) Suppose all bankers are ex-ante identical.*

$$\begin{aligned}
\underline{N}_{uv} = \underline{N} &:= \max\left\{0, \frac{\pi_C^{01}}{\rho_C} - \frac{\pi_O^{01}}{\rho_O}\right\}^{-1} \left(\pi_O^{01} - \pi_C^{01}\right) (\beta_{01} + \beta_{10}) \left(\frac{\sigma_{10}}{\xi} - 1\right) \\
\overline{N}_{uv} = \overline{N} &:= \frac{\rho_O}{\pi_O^{01}} \left(\pi_C^{01} (\beta_{01} - \beta_{10}) - \left(\pi_O^{01} - \pi_C^{01}\right) (\beta_{01} + \beta_{10}) \left(\frac{\sigma_{10}}{\xi} - 1\right)\right)
\end{aligned}$$

$\overline{N}$ does not depend on ρ_C and $\rho_O \delta^{01} > \overline{N}$. Additionally

$$\begin{aligned}
\mathcal{O}_{uv}^{01} = \mathcal{O}_u &:= \frac{1}{2Z\psi_A c_u}(((\delta^{01} \left(\overline{k}_u - \Lambda_u(\rho_O \delta^{01})\right) + (2\rho_O)^{-1} \left(\left(\Lambda_u(\rho_O \delta^{01})^2 - \Lambda_u(\max\{\overline{N}, \rho_C \delta^{01}\})^2\right)^+ + \left(\Lambda_u(\min \right.\right. \\
\mathcal{C}_{uv}^{01} = \mathcal{C}_u &:= \frac{1}{2Z\psi_A c_u}(((\delta^{01} \left(\Lambda_u(\overline{N}) - \Lambda_u(\rho_C \delta^{01})\right)^+ + (2\rho_C)^{-1} \left(\Lambda_u(\rho_C \delta^{01})^2 - \Lambda_u(\underline{N})^2\right)^+)))
\end{aligned}$$

Proof. **(Lemma 7)** (Auxiliary) Corollary of Lemma 6. The only nontrivial part is that

$$\begin{aligned}
&\rho_O \delta^{01} > \overline{N} \\
&\iff \rho_O \delta^{01} > \frac{\rho_O}{\pi_O^{01}} \left(\pi_C^{01} (\beta_{01} - \beta_{10}) - \left(\pi_O^{01} - \pi_C^{01}\right) (\beta_{01} + \beta_{10}) \left(\frac{\sigma_{10}}{\xi} - 1\right)\right) \\
&\iff 1 > \frac{\pi_C^{01}}{\pi_O^{01}} - \left(1 - \frac{\pi_C^{01}}{\pi_O^{01}}\right) \frac{(\beta_{01} + \beta_{10})}{\delta^{01}} \left(\frac{\sigma_{10}}{\xi} - 1\right)
\end{aligned}$$

which always holds. $\square$

Proof. **(Lemma 5)** Since bankers are ex-ante identical, $\mathbb{I}_{uv}^{qq'} = 1 \iff \sigma_{vq'} - \sigma_{uq} > 0 \iff q' = 1 \wedge q = 0$. Simply insert this $\mathbb{I}_{uv}^{qq'}$ this and insert $\mathcal{O}_{uv}^{01}, \mathcal{C}_{uv}^{01}$ from Lemma 7 into $e_{uv}^{**}, e_{uCv}^{**}$ in Lemma 6 to get the result. $\square$

Lemma 8. *(Auxiliary)* $\Gamma_{OP} < \frac{\delta}{\Delta+\delta}\Gamma_{CR} \iff \overline{N} > \rho_C \delta^{01} \iff \rho_C \delta^{01} > \underline{N}$.

Proof. **(Proposition 11)** By Proposition 10 and Lemma 5,

$$e_v^{\rightarrow} \overline{\omega}^2 = \pi_O^{01} \mathcal{O} \sum_u \left(\mu_{uv}^{01} - \mu_{vu}^{01}\right)^+ + \pi_C^{01} \mathcal{C} \left(\sum_u \left(\mu_{uv}^{01} - \mu_{vu}^{01}\right)\right)^+$$

Notice $\sum_v \sum_u (\mu_{uv}^{01} - \mu_{vu}^{01}) = 0$ and so $\min_v \sum_u (\mu_{uv}^{01} - \mu_{vu}^{01}) \leq 0$. Also by out-regularity, $\sum_u (\mu_{uv}^{01} - \mu_{vu}^{01})^+ = M\mu^*$ for all v. Then $\min e_v^{\rightarrow} \overline{\omega}^2 = \pi_O^{01} \mathcal{O} M \mu^*$. $\square$

Proof. **(Corollary 3)** Novation gap is empty. Then by algebra $\mathcal{O} = \delta^*$ and $\tilde{\mathcal{O}} = \delta^*$. Then $\text{Eff}_{O/C} = 0$. □

Proof. **(Corollary 4)** In this case, $\mathcal{O} = (4\rho_O Z\psi_A c)^{-1}\left\langle \overline{k}^2 - [\![\overline{N}]\!]^2\right\rangle = (2\rho_O)^{-1}(\overline{k}-\underline{k})\left\langle \overline{k}^2 - [\![\overline{N}]\!]^2\right\rangle$, $\tilde{\mathcal{O}} = (2\rho_O)^{-1}\left\langle \overline{k} - [\![\overline{N}]\!]\right\rangle^{-1}\left\langle \overline{k}^2 - [\![\overline{N}]\!]^2\right\rangle$. This is $(2\rho_O)^{-1}\left(\overline{k} + [\![\overline{N}]\!]\right)$ if $\overline{k} - [\![\overline{N}]\!] > 0$. $\tilde{\mathcal{O}}$ is not well-defined otherwise.

For $\overline{N} < \underline{k}$, there is no novation. $\tilde{\mathcal{O}} = \mathcal{O} = (2\rho_O)^{-1}\left(\overline{k} + \underline{k}\right)$, $\text{Eff}_O = 1$, $\text{Eff}_C = 0$. For $\overline{N} > \overline{k}$, all contracts are novated. There is no coordination failure. Eff is not well-defined.

For $\underline{k} < \overline{N} < \overline{k}$, recall $\overline{N} = \rho_O\left(\delta(1-\Gamma_{\text{OP}}) - \Gamma_{\text{OP}}\Delta\right)$. Then

$$\begin{aligned}
2\tilde{\mathcal{O}} &= (\rho_O)^{-1}\left(\overline{k} + \overline{N}\right) = (\rho_O)^{-1}\left(\overline{k} + \rho_O\left(\delta(1-\Gamma_{\text{OP}}) - \Gamma_{\text{OP}}\Delta\right)\right) \\
&= \rho_O^{-1}\overline{k} + \delta(1-\Gamma_{\text{OP}}) - \Gamma_{\text{OP}}\Delta
\end{aligned}$$

Then $\text{Eff}_C = 0$ and

$$\text{Eff}_O = \frac{\rho_O \rho_O^{-2}\overline{k}}{\rho_O^{-1}\overline{k} + \delta(1-\Gamma_{\text{OP}}) - \Gamma_{\text{OP}}\Delta} = \frac{\overline{k}}{\overline{k} + \overline{N}} \in (\frac{1}{2}, 1)$$

□

Proof. **(Corollary 5)** In this case, $\mathcal{O} = (4\rho_O Z\psi_A c)^{-1}\left\langle [\![\underline{N}]\!]^2 - \underline{k}^2\right\rangle = (2\rho_O)^{-1}(\overline{k}-\underline{k})\left\langle [\![\underline{N}]\!]^2 - \underline{k}^2\right\rangle$, $\tilde{\mathcal{O}} = (2\rho_O)^{-1}\langle [\![\underline{N}]\!] - \underline{k}\rangle^{-1}\left\langle [\![\underline{N}]\!]^2 - \underline{k}^2\right\rangle$. This is $(2\rho_O)^{-1}([\![\underline{N}]\!] + \underline{k})$ if $[\![\underline{N}]\!] - \underline{k} > 0$. $\tilde{\mathcal{O}}$ is not well-defined otherwise.

For $\underline{N} > \overline{k}$, there is no novation. $\tilde{\mathcal{O}} = \mathcal{O} = (2\rho_O)^{-1}\left(\overline{k} + \underline{k}\right)$, $\text{Eff}_O = 1$, $\text{Eff}_C = 0$. For $\underline{N} < \underline{k}$, all contracts are novated. There is no coordination failure. Eff is not well-defined.

For $\underline{k} < \underline{N} < \overline{k}$, there is novation. Then $\underline{N} < \overline{k} < \rho_C\delta^*$, in particular $\underline{N} < \rho_C\delta^*$. Then $\frac{\Gamma_{\text{OP}}}{\Gamma_{\text{CR}}} < \frac{\delta^*}{\Delta+\delta^*}$ by Proposition 11. Then $\Gamma_{\text{OP}} < \Gamma_{\text{CR}}$. Then $\underline{N} = \rho_C\left(\Gamma_{\text{CR}} - \Gamma_{\text{OP}}\right)^{-1}\Gamma_{\text{OP}}\Delta = \frac{\Gamma_{\text{OP}}\Delta}{\frac{1-\Gamma_{\text{OP}}}{\rho_C} - \frac{1}{\rho_O}}$. Then

$$\begin{aligned}
2\tilde{\mathcal{O}} &= (\rho_O)^{-1}\left(\underline{N} + \underline{k}\right) = (\rho_O)^{-1}\left(\rho_C\left(\Gamma_{\text{CR}} - \Gamma_{\text{OP}}\right)^{-1}\Gamma_{\text{OP}}\Delta + \underline{k}\right) \\
&= \frac{(1-\Gamma_{\text{OP}})\,\Gamma_{\text{OP}}\Delta}{1-\Gamma_{\text{OP}} - \frac{\rho_C}{\rho_O}} + \underline{k}\frac{1}{\rho_O} \\
\Longrightarrow \text{Eff}_C &= \frac{\rho_C}{\underline{N}+\underline{k}}\frac{d\underline{N}}{d\rho_C} = \frac{\rho_C}{(\underline{N}+\underline{k})}\frac{\underline{N}^2}{\Gamma_{\text{OP}}\Delta}\frac{1-\Gamma_{\text{OP}}}{\rho_C^2} = \frac{\underline{N}}{\underline{N}+\underline{k}}\frac{1-\Gamma_{\text{OP}}}{\Gamma_{\text{CR}} - \Gamma_{\text{OP}}} \\
\text{and} \text{Eff}_O &= -\rho_O\left(\frac{1}{\underline{N}+\underline{k}}\frac{d\underline{N}}{d\rho_O} - \frac{1}{\rho_O}\right) = \rho_O\frac{1}{\underline{N}+\underline{k}}\frac{\underline{N}^2}{\Gamma_{\text{OP}}\Delta}\frac{1}{\rho_O^2} + 1 \\
&= \frac{\underline{N}}{\underline{N}+\underline{k}}\frac{1-\Gamma_{\text{CR}}}{\Gamma_{\text{CR}} - \Gamma_{\text{OP}}} + 1
\end{aligned}$$

□