

Network Reactions to Banking Regulations[☆]

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Abstract

Optimal regulatory restrictions on banks have to solve a delicate balance. Tighter regulations reduce the likelihood of banks' distress. Looser regulations foster the allocation of funds towards productive investments. With multiple banks, optimal regulation becomes even more challenging. Banks form partnerships in the interbank lending market in order to face liquidity needs and to meet investment possibilities. We show that the interbank network can suddenly collapse when regulations are pushed beyond a critical level, with a discontinuous increase in systemic risk as the cross-insurance of banks collapses.

Keywords: banking regulations, interbank networks, systemic risk.

1. Introduction

Banks provide a key intermediation function in the economy. They extend loans to agents with productive investment opportunities using funds from depositors who do not have access to those opportunities. Because banks' investments can fail, depositors are only willing to provide the funds if they are ex-ante compensated for potential losses, which forces bankers to internalize failure and to invest efficiently, both in terms of scale and risk exposure. The scenario changes, however, in the presence of governments that have ex-post incentives to cover the losses of depositors with distortionary taxation proceedings. These bailouts provide an implicit insurance for depositors that allows banks to obtain funds at a subsidized rate and invest those funds excessively, either at a larger

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scale or by taking more risks, relative to the efficient allocation. As highlighted by Nosal and Ordonez (2016), when the government lacks commitment not to bail out depositors in distress, banks tend to invest excessively and inefficiently, which increases the likelihood and the magnitude of crises and the need for distortionary bailouts.

In response to this time inconsistency governments tend to regulate banking activity, in part to counteract these perverse ex-ante banks' incentives. The banking regulations that are introduced to restrict the volume and the risk of investments take several forms. Capital requirements specify the level of equity that a bank must hold as a percentage of its risk-weighted assets. Liquidity and reserve requirements impose a direct upper bound on the amount of loans that can be extended by a bank per U.S. dollar kept physically in possession of the bank. Credit rating requirements curb the choice of projects that a bank can choose to maintain a certain credit qualification. Financial disclosure requirements increase the cost of extending and managing loans. Even though all these restrictions are introduced to tackle different situations, all have the consequence of restricting the volume and riskiness of a bank's assets.

Too tight regulations reduce excessive investments and risk-taking and, thus, the need for bailouts, but they may end up choking the bank's ability to channel funds to productive investment opportunities. When these two forces change smoothly on the level of regulatory requirements for an individual bank, it should in principle be possible to obtain the optimal level of regulation that counteracts the banks' incentives to extend excessive (and excessively risky) loans.

Although this logic is sound in the case of a single bank, it becomes more challenging in the case of multiple banks that interact with one another. Indeed, banks do not operate in isolation. After extending loans for a lucrative investment opportunity, a bank may fall short of the liquidity needed to cover potential extra refinancing needs to continue old projects or to take advantage of new profitable projects. When this occurs they refer to other banks for short-term loans in order to satisfy these needs. This implies that the tightness of regulatory constraints may have a first-order impact on the vibrancy and interconnectedness of the interbank market. How does the interbank network react to changes in regulation? How should regulation be determined in a setting in which banks interact? How does the endogenous formation of interbank networks affect the welfare effects of regulation?

In this paper we explore these questions and show that the topology and the level of interconnectedness of the interbank network react discontinuously to regulatory requirements, adding an additional layer of complexity to the delicate balance that optimal banking regulations should strive to achieve. The reaction of an interbank network features a "phase transition": beyond a tipping point of regulatory requirements, the network becomes disproportionately less interconnected, with systemic risk increasing discontinuously in response to this abrupt change in the network architecture.

To fix ideas we will focus on a specific type of regulatory constraint: liquidity requirements. On the one hand, the coverage of liquidity shortages is one of the

main functions of interbank lending, constituting the blood of banking networks' operations. On the other hand, a micro-founded model encompassing regulation in general would require many more elements than those needed to show the deep source of network fragility: restrictions on the volume of loans. Even though we craft our model to specifically capture restrictions on the ratio of liquid to illiquid assets, we map our results on to other banking regulations and show that we should expect the same fragility from them.

In our model, the benefit of having an interbank counterparty is that it provides additional "insurance" in case that a refinancing shock realizes, which requires extra funds to continue or expand some of a bank's investments. Even though some of the bank's counterparties are expected to become insolvent, the rest can provide extra liquidity should a refinancing shock occur. On the one hand, the marginal benefit of having an extra counterparty decreases as liquidity requirements tighten up because the loan sizes decline and banks obtain less benefits from insuring continuation of those projects. On the other hand, we assume that the marginal cost of an extra counterparty is independent of the liquidity requirement or the investment scale.

We demonstrate that as liquidity requirements tighten, not only does the desired level of counterparties decrease, but after a critical point a bank discontinuously prefers to reduce its counterparties. When that happens the network structure changes suddenly from very dense to very sparse and the aggregate level of interbank activity collapses. In other words, as liquidity requirements tighten not only investment declines but also systemic risk – measured by the fraction of banks that choose to close operations – increases, discontinuously after a certain threshold. This sudden change induces a discontinuous increase in distortionary bailouts and a discontinuous decline in welfare.

The logic behind the fragility of the network structure relies on the strategic considerations of a bank when it chooses its level of connectivity. When a bank chooses how many counterparties to have, it takes into account how many of those counterparties are expected to be able to provide assistance in the case of a refinancing need. How many of a bank's counterparties will be able to provide assistance, however, depends on the number of counterparties that each of the bank's counterparties choose to have. With many banks these strategic considerations may in principle be intractable. Our model describes the conditions under which a unique stable network is possible, and this allows us to develop comparative statics.

Creating a banking network model that puts these strategic considerations at the forefront of the discussion is relevant for two reasons. First, connectivity decisions of other banks have a large impact on each bank's connectivity decisions, and these strategic considerations create a highly nonlinear network response to changes in regulation. Second, a bank does not internalize the effects of its own connectivity decisions on others which can create severe externalities.

Our result also has important policy implications. The highest welfare that takes into account endogenous network formation tends to be achieved at high levels of connectivity and close to the tipping point at which networks discontinuously collapse. In principle this calls for setting regulations close to the tipping

point that sustains dense networks. This is, however, a dangerous endeavor when it is carried out in the presence of exogenous shocks to fundamentals that can change the network’s tipping point. Setting regulatory requirements very close to the tipping point increases the likelihood of a collapse, and, thus, a crisis. This suggests that the optimal distance between regulatory requirements and the tipping point should balance the loss in welfare from a lower level of connectivity and the gains from a lower probability of a crisis.

In order to evaluate the effect of banking regulations on networks we compare the liquidity requirements chosen by a *network-conscious* regulator who understands the reaction of interbank insurance possibilities to changes in regulation and a *network-blind* regulator who does not understand that networks change with regulation. We show that the potential welfare losses that occur when network reactions (including discontinuous collapse) are ignored can be large.

Literature Review: The recent financial crisis has been a catalyst for scholarly research into both banking regulations and interbank networks. Although there is a recent rich literature on these two topics, our paper is one of the few, if not the first, to combine these two policy-relevant topics and analyze the effects of regulatory requirements on interbank network formation.

With regards to banking regulations, most of the literature has focused on the optimal level of a *single bank’s* liquidity requirement to prevent bank runs. See, for example, Cooper and Ross (1998), Ennis and Keister (2006), Calomiris et al. (2014), Santos and Suarez (2015), Diamond and Kashyap (2016) and van den Heuvel (2016). In spite of these efforts, Allen (2014), who surveys the recent literature on liquidity regulation, concludes that “much more research is required in this area. With capital regulation there is a huge literature but little agreement on the optimal level of requirements. With liquidity regulation, we do not even know what to argue about.” Confusion about how liquidity requirements help facing liquidity shortages comes in large part from the lack of consensus of what a liquidity shortage is. While the literature cited above focuses on using liquidity regulation to curb the effects of liquidity shocks on banks’ short-term liabilities as in Diamond and Dybvig (1983), our paper focuses on using liquidity regulations to curb the effects of liquidity shocks on banks’ assets refinancing needs as in Holmström and Tirole (1998).

A more recent literature acknowledges banking interconnections when studying liquidity regulations. Farhi et al. (2009) study the role of liquidity requirements when the liquidity of one bank reduces the likelihood of a run in another bank. Wang (2016) highlights the externalities that characterize the formation of linkages and the importance of conditioning regulation on the topology of banking networks. Aldasoro et al. (2015) also discuss the effects of banking regulations in the presence of banking interconnections. They show that liquidity requirements decrease systemic risks at the cost of lower efficiency given a network structure. By endogenizing the network reaction, we show instead that there exists a critical point beyond which liquidity requirements both increase systemic risks and reduce efficiency discontinuously.

There is also a heated debate about liquidity requirements that are alternatives to standard reserve requirements, such as the liquidity coverage ratio (LCR) and the net stable funding ratio (NSFR) that have been proposed by the Dodd Frank Act and by the Basel Committee on Banking Supervision. The LCR extends the interpretation of “liquid assets” beyond reserves, such as assets with higher interest rates. Goodfriend (2016) has examined the high costs, complexity, and discretion vulnerability of LCR relative to standard reserve requirements. The NSFR defines the portion of capital and reliable liabilities that financial intermediaries have to maintain over a specified time horizon relative to the amount of required stable funding during that time period. Although our setting is constructed to capture reserve requirements, it could accommodate these two alternatives by including more assets classes (LCR) or a more involved timing of funding needs (NSFR).

Not surprisingly then, there are not many empirical studies of the effects of policies on banking networks. An exception is Paddrik et al. (2016), who show that the pattern of reserve requirements established by the National Banking Acts (NBA) in the U.S. dramatically changed the network structure of interbank deposits between 1862 and 1867 in Pennsylvania. This involved a reinforcement of linkages between Philadelphia and New York banks and a weakening of the linkages between county banks. Their finding is consistent with our model.

Another recent literature, such as Erol and Vohra (2016), examines endogenous networks, but these are not systematic studies that seek to understand the endogenous reaction of financial networks to changes in banking regulations and their effects on systemic risk and welfare. Allen and Gale (2000) discuss bankruptcy contagion in an exogenous network (modeled as a ring) and, consistent with our results, show a negative relation between the degree of connectivity and systemic risk. Battiston et al. (2012) show that the relationship between connectivity and systemic risk can be hump-shaped, and they demonstrate that this is so because low connectivity implies less risk-sharing and high connectivity implies more exposure to the default of partners. Acemoglu et al. (2015) also show that the effects of connectivity can go in both directions for different magnitudes of shocks. In a setting with heterogenous banks, Farboodi (2015) also provides a model of banking networks, within which funds flow from savers to investment opportunities, possibly over several links, and she demonstrates that the distribution of surplus depends on the network topology. None of these papers, however, studies the reaction of networks to changes in policy.

Finally, in contrast to Ordonez (2016), who studies the unforeseen effects of banking regulations on financial innovations that allow banks to channel activities outside the scope of regulators (the so-called shadow banking), this paper focuses on the unforeseen effects of banking regulations on networks, specifically the creation and destruction of banking linkages, that affect both systemic risk and welfare. We contend that the unforeseen effects of banking regulations should be included in discussions of optimal regulations.

We structure this paper as follows. In the next section we present a simple model that includes liquidity requirements and the possibility that linkages can be created with other banks in order to face refinancing needs. In Section 3

we explain the main forces in the model and show the characterization of the network structure for a given level of regulation. In Section 4 we study the effects of liquidity requirements on the anatomy of a banking network. In Section 5 we characterize optimal regulation by a network-conscious regulator who factors in network effects. In Section 6 we compute the welfare losses of a network-blind regulator who does not take into account the effects of regulations on networks. We conclude with some final remarks and considerations.

2. Model

2.1. Environment

We consider a single-period economy composed of k risk-neutral banks, k risk-neutral households and a government. We denote by $\mathcal{N} = \{n_1, \dots, n_k\}$ the set of banks. Each bank n_i has access to a unique project (of maximum possible scale, \bar{L}_i) to finance, and each is associated with a unique household n_i^h . We assume that the household n_i^h deposits D_i in bank n_i at the beginning of the period and wishes to withdraw and consume at the end of the period.

The timing of actions and events within the period can be split into five stages. First is a *regulation stage*. The government sets a liquidity requirement $\phi \in [0, 1]$, which imposes a minimum ratio of liquid to illiquid assets that a bank has to hold on its balance sheet. The role of the policy parameter ϕ is to restrict the bank's investments in risky and illiquid assets. At the end of the section we discuss how ϕ maps on to policies that we observe in reality, such as reserve requirements, different types of liquidity requirements, capital requirements, and more.

Second is a *network formation stage*. By mutual consent banks form links that serve as credit lines that insure one another against future refinancing shocks that could prevent projects from reaching maturity. A link between banks n_i and n_j is denoted $l_{ij} = l_{ji}$, and the resulting set of links is denoted $\mathcal{L} \subset [\mathcal{N}]^2$. Then, $(\mathcal{N}, \mathcal{L})$ is the realized interbank network. Denote $\mathcal{N}_i = \{j : l_{ij} \in \mathcal{L}\}$ the set of counterparties of n_i and $d_i = |\mathcal{N}_i|$ its degree (the number of counterparties). Forming a link involves a utility cost κ_l to each counterparty.

Third is an *investment stage*. Each bank n_i extends in checks a loan of L_i to finance the project, keeping its deposits D_i as reserves. The liquidity requirement restricts the size of the loan (and thus the scale of the project) to L_i , as $D_i \geq \phi L_i$, where $L_i \leq \bar{L}_i$ is the natural limit. Notice that when investing L_i total assets are $D_i + L_i$ (D_i in cash and L_i in loans) while total liabilities are also $D_i + L_i$ (D_i as debt to the households, or "original depositors," and L_i is debt to check holders, or "business depositors").³ The returns from the

³The project uses bank n_i 's checks to operate - for example to buy raw materials or to pay workers. The recipients of these checks, for a total of L_i , eventually deposit them in the bank, at which point they become "business depositors." The assumption that a bank uses checks to finance its investment fully is an optimal bank's response when the project displays constant returns to scale and its return is higher than the return from holding cash. Under

projects materialize at the end of the period. Bank deposits and checks also are due at the end of the period.

Fourth is a *continuation stage*. The projects managed by all banks are publicly revealed to be $\theta_i \in \{B, G\}$, where B represents a *bad project* that never matures and G a *good project* that can succeed if appropriate actions are taken by the bank in the next stage. These types are independent across banks, with G happening with probability α_i and B happening with probability $1 - \alpha_i$.

After the projects' types are publicly observed, each bank n_i chooses an action a_i . Banks managing good projects can choose to continue (C) operating the project, or not (N), then $a_i \in \{C, N\}$. Continuing operation incurs a management cost (an effort cost) of κ_c per unit of investment and κ'_i to maintain the counterparty, making up a total of $\kappa_c L_i + \kappa'_i d_i$. Banks that have bad projects always choose $a_i = N$; that is, the bank would only pay the previous continuation costs but would not recover any payoffs from the project. Let g_i and f_i respectively denote the number of counterparties of n_i that chose to continue and not continue: $g_i = |\{n_j \in \mathcal{N}_i : a_j = C\}|$ and $f_i = |\{n_j \in \mathcal{N}_i : a_j = N\}|$.

A bank n_i that experiences a good shock and chooses N liquidates its project early and recovers $R_1 L_i$ where $1 > R_1 \geq 0$. This implies that the total assets of a non-continuing bank at the end of the period are $D_i + R_1 L_i$ while its total liabilities are $D_i + L_i$. We assume that the government faces a large (maybe political) disutility from depositors who do not get their funds back at the end of the period. This simple assumption guarantees that the government would always bail out the defaulted depositors. The funds used for bailouts per bank in need at this stage amount to $(1 - R_1)L_i$. Banks n_i that chooses not to continue then receive 0 payoff.

A bank n_i that chooses C and continues business moves to a fifth and last *refinancing stage*. At that stage no more than one of the projects of the continuing banks receives a refinancing shock, which is characterized by new funds that are needed in cash for the project to mature (a liquidity shock). The probability that a bank n_i , and only n_i , needs extra funds is η_i . With probability $\eta_0 = 1 - \sum_i \eta_i$ no project receives a refinancing shock. Conditional on the continuing bank n_i receiving a refinancing shock, the amount of funds needed is ρ_i , drawn from a distribution with c.d.f. F_i . Investors can obtain these extra funds only from their associated counterparty banks in the network and no others.

The banks that do not face a refinancing shock and that have invested at scale L_i obtain a return $R_2 L_i$, where $R_2 > 1$. As for the bank facing the refinancing need, if the bank obtains enough funds to refinance the liquidity need ρ_i , then the project's return scales by $m > \frac{1}{R_2}$ and pays an extra ρ_i , at which point the return to n_i is $m R_2 L_i + \rho_i$. If the bank does not obtain enough funds to refinance the liquidity need, the project fails and the return to n_i is 0.

these circumstances the bank always chooses to lever as much as possible, with the amount of liquid assets given exogenously by D_i . A model with an interior optimal portfolio choice is potentially interesting, but is not only beyond the scope of the paper but also complicates the network analysis considerably.

Finally, we assume that after the refinancing shock is observed, each household n_i^h receives extra funds (say wages) W_i , which are deposited at bank n_i if n_i has continued and is still in business. Then, without interbank activity, a bank n_i that faces a refinancing shock relies on its own available funds $W_i + D_i$ to ride the shock. This implies that under these conditions the bank would be able to refinance the project to completion if and only if $\rho_i \leq W_i + D_i$.

Banks, however, can engage in interbank lending through credit lines (links) formed in advance during the network formation stage. We assume that funds do not travel further than one link, which is the main friction in the interbank market; in other words, a bank intermediates not between two banks but between one household and one bank. Notice that due to the extra ρ_i return on top of mR_2L_i , there is no risk in lending these funds to the bank that faces a refinancing need; however this is conditional on knowing that n_i will be able to borrow $\rho_i - D_i - W_i$ in total from its counterparties. A counterparty n_j , if it has decided to continue, has W_j excess liquidity to lend to n_i .⁴ Therefore, n_i can cover ρ_i if and only if $\rho_i \leq D_i + W_i + \sum_{j:n_j \in \mathcal{N}_i, a_j=C} W_j$.

The timeline in Figure 1 summarizes this sequence of events and actions as well as the main notation we have introduced.

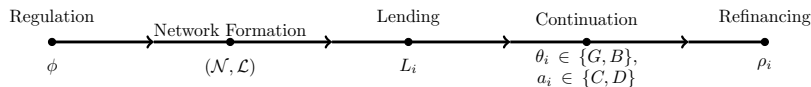


Figure 1: Timeline of events

2.2. Interpretation of the policy parameter ϕ .

Notice that the parameter that determines the extent of banking regulation in our setting is ϕ , which imposes an upper bound on the ratio of liquid assets to illiquid assets, which is $\frac{D_i}{L_i} \geq \phi$. While liquid assets D_i is meant to include cash, treasury bills, repos, central bank reserves, and any other asset that can be converted easily and quickly into cash, illiquid assets L_i include loans, mortgages and other investments whose liquidation is costly and incurs a loss. Here we show that, when referring to a tightening in regulation ϕ , we also capture a tightening in other diverse forms of banking regulatory requirements.

Reserve requirements, for example, are defined by the Federal Reserve Bank as “the amount of funds that a depository institution must hold in reserve against specified deposit liabilities.” In our model reserves are captured by D_i and total liabilities by $D_i + L_i$, then reserve requirements are given by a constraint $\frac{D_i}{D_i + L_i} \geq \hat{\phi} \equiv \frac{\phi}{1 + \phi}$. This implies that an analysis of changes in ϕ is isomorphic to an analysis of changes in reserve requirements $\hat{\phi}$.

⁴Motivated by the use of reserve requirements, banks are only allowed to lend above required reserves in the interbank market. Hence only W_j can be used on the interbank market. This can be rationalized as the limit of a credit line. This assumption allows us to simplify the optimal choice of L_i which is otherwise intractable due to network externalities.

Similarly, and although a liquidity requirement is conceptually a constraint on the ratio of liquid to illiquid assets, as is the case in our model, in reality these constraints take several forms. The liquidity coverage ratio (LCR), for example, is defined by the Federal Reserve Bank as “*the amount of high quality, liquid assets (HQLA) such as central bank reserves and government and corporate debt that can be converted easily and quickly into cash that has to be equal to or greater than its projected cash outflows minus its projected cash inflows during a 30-day stress period.*” Assuming that in our model the expected net cash outflow during a stress period is a fraction γ of total liabilities, then the LCR is given by a constraint $\frac{D_i}{\gamma(D_i+L_i)} \geq \tilde{\phi} \equiv \frac{\phi}{\gamma(1+\phi)}$. Again, changes in ϕ are isomorphic to changes in LCR $\tilde{\phi}$.

Other forms of liquidity requirements applied by regulators include the net stable funding ratio (NSFR), which recently has been applied. The NSFR, which specifies a more involved timing of funding needs and definition of applicable assets, changes how the γ above is defined. From the perspective of our model, however, an increase in ϕ has the same effect as an increase in the NSFR.

Another important banking regulation is given by capital requirements, which is defined as “*the amount of capital a bank or other financial institution has to hold as required by its financial regulator, usually expressed as a capital adequacy ratio of equity that must be held as a percentage of risk-weighted assets.*” Although we do not include bank capital in our model, imagine that instead of households depositing funds D_i they hold bank stocks for $E_i = D_i$, which banks then can use to invest in risky and illiquid assets, L_i . In this case, capital requirements can be expressed as a condition $\frac{E_i}{L_i} \geq \phi$, and the model is identical to the one we study here.

Finally, any other regulatory requirement that raises the costs of investing in illiquid assets more than it raises the costs of investing in liquid assets effectively reduces L_i relative to D_i and acts as a de facto upper bound (endogenously chosen by banks) on the ratio $\frac{D_i}{L_i}$. Examples of these regulations include credit rating requirements that put constraints on the riskiness of investments or the expected returns of loans and disclosure requirements that increase the cost of extending loans to non-standardized assets.

3. Banking Networks

We now solve for the equilibrium network. Working backwards we first solve the investment, continuation and refinancing stages. Then, based on these solutions, we discuss our solution concepts and characterize the equilibrium network (solving the network formation stage) as a function of an arbitrary regulatory parameter ϕ .

3.1. Investment, Continuation and Refinancing Stages

3.1.1. Payoffs

Refinancing stage: A bank n_i that does not continue in the continuation stage pays $\kappa_l d_i$ in terms of utility to form the network, has the ex-post income

$D_i + R_1 L_i$ and owes $D_i + L_i$. After paying all available assets to depositors, it obtains 0 and depositors receive a bailout for $(1 - R_1)L_i$. Then the bank's ex-post (utility) payoffs are $-\kappa_l d_i$.

If bank n_i continues and does not receive a refinancing shock, it pays back its depositors and obtains a return $(R_2 L_i + D_i + W_i) - (L_i + D_i + W_i) = (R_2 - 1)L_i$. The bank that continues also incurs utility costs to form the network, $\kappa_l d_i$, to maintain the network, $\kappa'_l d_i$ and to manage the project, $\kappa_c L_i$. Then the continuing bank's ex-post (utility) payoffs are $(R_2 - 1)L_i - (\kappa'_l d_i + \kappa_c L_i) - \kappa_l d_i$.

If the bank n_i continues and receives a refinancing shock, then payoffs depend on whether or not it can obtain funds to refinance. If the bank can refinance the shock ρ_i , it receives $mR_2 L_i - L_i$ ex-post payoffs because it uses the ρ_i extra return on top of $mR_2 L_i$ to repay the credit from counterparties. Notice that interbank lending does not enter a bank's payoffs since lending to a troubled bank only happens if repayment is certain. Then this bank's ex-post (utility) payoffs are $(mR_2 - 1)L_i - (\kappa'_l d_i + \kappa_c L_i) - \kappa_l d_i$. If the bank cannot refinance the shock ρ_i , it cannot fully repay its depositors and has ex-post (utility) payoffs $-(\kappa'_l d_i + \kappa_c L_i) - \kappa_l d_i$.

Continuation stage: Let M_i denote the expected net return of a bank n_i per unit of loan, net of the management costs of the network:

$$M_i = (1 - \eta_i)(R_2 - 1) + \eta_i F_i \left(D_i + W_i + \sum_{j: n_j \in \mathcal{N}_i, a_j = C} W_j \right) (mR_2 - 1) - \kappa_c \quad (1)$$

Accordingly, the bank n_i that continues has an expected payoff (at the end of the continuation stage)

$$\Pi(C) = M_i L_i - (\kappa'_l + \kappa_l) d_i. \quad (2)$$

If n_i does not continue, its payoff is $\Pi(N) = -\kappa_l d_i$.

Investment stage: At the continuation stage bank n_i will best respond, so that its payoff is $-\kappa_l d_i + \max\{0, M_i L_i - \kappa'_l d_i\}$. M_i is independent of L_i because the continuation decisions of n_i 's counterparties depend only on the amount that n_i can lend to them in the interbank market, which is W_i , independent of L_i . This makes the the expected payoff of n_i weakly increasing in L_i . Thus banks choose to extend the largest loan possible during the investment stage,

$$L_i = \min \left\{ \frac{D_i}{\phi}, \bar{L}_i \right\}.$$

3.1.2. Simplifying Assumptions

For the sake of simplicity in the exposition that follows we focus on a symmetric scenario in which all banks are identical. At the end of this section we discuss the implications of heterogeneity for network formation.

Assumption 1. $\bar{L}_i = \bar{L}$, $D_i = D$, $W_i = W$, $\alpha_i = \alpha$, $\eta_i = \eta$, $F_i = F$ for all n_i .

Henceforth we denote $L(\phi) = \min\{\frac{D}{\phi}, \bar{L}\}$ for all banks. Also denote $\beta = \eta(mR_2 - 1)$. Recall that the total number of counterparties of n_i that continue is g_i ; these are the banks that lend their excess liquidity to bank n_i when n_i needs to refinance its project. Moreover define $T(d_i|\phi)$ as the “probability threshold” of n_i for continuing. This is

$$T(d_i|\phi) = T_0 + \frac{d_i \kappa'_l}{\beta L(\phi)}, \quad T_0 = \frac{\kappa_c - (1 - \eta)(R_2 - 1)}{\beta} \quad (3)$$

Under Assumption 1 we can simplify the expression of the net return per unit of loan from (1) and rewrite the expected payoff from (2) as

$$\Pi(C, g_i, d_i|\phi) = -\kappa_l d_i + \beta L(\phi) [F(D + W + g_i W) - T(d_i|\phi)], \quad (4)$$

$$\Pi(N, g_i, d_i|\phi) = -\kappa_l d_i \quad (5)$$

in the case of continuation and not-continuation, respectively.

If $T(d_i|\phi) > 1$, n_i plays N regardless, as $F(\cdot) \leq 1$. Similarly, if $T(d_i|\phi) < 0$ n_i plays C regardless, as $F(\cdot) \geq 0$. For $T(d_i|\phi) \in (0, 1)$, define the *fragility* of a bank n_i with degree d_i as

$$S(d_i|\phi) \equiv \lceil (F^{-1}(T(d_i|\phi)) - D - W)/W \rceil - 1.$$

This measure of fragility captures the number of successful counterparties a bank needs if it is to refinance the project. As fragility S increases so must the success of the counterparties that a bank needs if it is to continue. More specifically, as shown in Figure 2, a bank n_i plays N if $g_i \leq S(d_i|\phi)$; otherwise it plays C .⁵

Conditional on the total number of counterparties and the number of counterparties that continue, a bank n_i is more likely to continue when $S(d_i|\phi)$ declines, which occurs when the bank has more funds on its own (i.e. a higher D and W) or when the threshold for refinancing decreases and the bank has more incentives to continue (i.e. a higher $T(d_i|\phi)$). The latter happens when the net present value of the project is relatively large (high R_2), when the cost of continuation is relatively small (a low κ_c or κ'_l), or when the probability that the project needs refinancing is relatively small (a low η).

For expositional simplicity we assume that F is uniform on $[0, P]$, which implies that $F(D + W + gW) = \max\{0, \min\{1, \frac{D+W+gW}{P}\}\}$. Moreover, we assume that the largest possible refinancing need P cannot be met at any level of connectivity. The latter ensures that every additional counterparty that continues strictly improves the expected return.

Assumption 2. $F \sim U[0, P]$, $P > D + W + kW$.

Before moving to solving for the equilibrium network, we specify the solution concepts.

⁵Here we follow the convention that $F^{-1}(t) = \infty$ for $t > 1$ and $-\infty$ for $t < 0$.

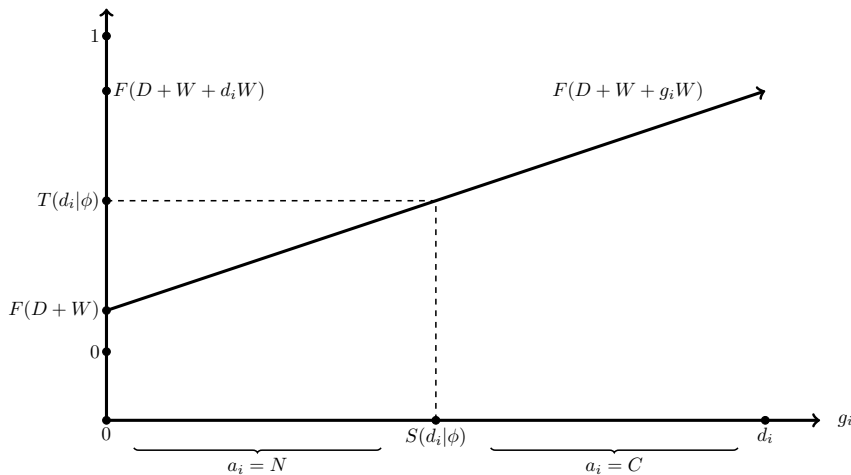


Figure 2: The decision to continue or not conditional on d_i and g_i

3.2. Solution Concepts

In the continuation stage, banks that experience bad shocks are forced to play N . Banks that experience good shocks play a binary game among each other. This game is supermodular. The solution concept is the cooperating equilibrium: the Nash equilibrium in which the largest set of agents, with respect to set inclusion, play C among all Nash equilibria. Due to supermodularity, this equilibrium notion is well-defined. Supermodularity, according to Tarski's Theorem, implies that the set of Nash equilibria is a complete lattice, and the cooperating equilibrium is the highest element of the lattice.

An alternative definition of the cooperating equilibrium can be given via a strategic contagion argument, which employs a natural interpretation in financial contagion setup. Banks that receive bad shocks are forced to discontinue. After some banks become insolvent in this way, some solvent banks that are tightly connected to insolvent banks now are more likely to be illiquid: they have less likelihood of refinancing their projects and prefer not to continue in order to save costs. These banks find N iteratively strictly dominant, and so on. This highlights the strategic aspect of contagion in our model. The iterated elimination of strictly dominated strategies resembles a black-boxed financial contagion. Along the iteration, at the point that the remaining banks can rationalize C , the contagion stops. The resulting profile is the rationalizable strategy profile in which anyone who can rationalize C do play C . Supermodularity ensures that this profile is also a Nash equilibrium.

During the network formation stage, banks evaluate each network with their expected payoffs (over the shocks θ 's) in the cooperating equilibrium in the subsequent periods. Agents form a strongly stable network that is defined as

follows. Consider a candidate interbank network $(\mathcal{N}, \mathcal{L})$ and a subset \mathcal{N}' of agents. A feasible deviation by \mathcal{N}' from \mathcal{L} is one in which i) \mathcal{N}' can add any missing links or cut any existing links that stay within \mathcal{N}' , and ii) \mathcal{N}' can cut any of the links between \mathcal{N}' and \mathcal{N}/\mathcal{N}' . A profitable deviation by \mathcal{N}' from \mathcal{L} is a feasible deviation in which the resulting network yields a strictly higher expected payoff to every member of \mathcal{N}' . An interbank network $(\mathcal{N}, \mathcal{L})$ is strongly stable if there are no subsets of \mathcal{N} with a profitable deviation from \mathcal{L} .⁶

The notion of network formation that we employ embeds many strategic concerns regarding the formation of links. When choosing the number of counterparties, each bank has to infer how many of them will be able to help in case of a refinancing shock, which, in turn, depends on the number of counterparties that each counterparty chooses. Moreover, multiple banks can form coalitions that coordinate their decisions regarding their links with other banks, and this models another layer of strategic sophistication among banks.

3.3. Equilibrium Network

Payoff functions $\Pi(C)$ in (4) and $\Pi(N)$ in (5) are special cases of Erol (2016), who proves the existence and uniqueness of strongly stable networks in a more general setup.

Define the functions V and d^* as follows.

$$\begin{aligned} V(d|\phi) &:= \mathbb{E}_{\tilde{g}}[(1 - \alpha) \times \Pi(N, \tilde{g}, d|\phi) + \alpha \times \max\{\Pi(C, \tilde{g}, d|\phi), \Pi(N, \tilde{g}, d|\phi)\}] \\ &= -\kappa_l d + \alpha\beta L(\phi) \mathbb{E}_{\tilde{g}}[\max\{0, F(D + W + \tilde{g}W) - T(d_i|\phi)\}] \end{aligned}$$

where $\tilde{g} \sim G[\cdot, d, \alpha]$ for G is defined as the c.d.f. of binomial distribution with d trials and α success probability. V can be thought of as a hypothetical payoff function for a single bank with degree d that supposes its counterparties default if and only if they received bad shocks. In other words, V is the payoff function absent contagion. Define $d^{**}(\phi) := \operatorname{argmax}_{d \in \mathbb{Z}^+} V(d|\phi)$ as the hypothetical optimal degree for a single bank if there was no contagion. Since $F(\cdot) \leq 1$, d^{**} is generically well-defined. As there is an upper bound on the number of counterparties, the relevant optimal degree is

$$d^*(\phi) := \operatorname{argmax}_{d \leq k-1} V(d|\phi).$$

Clearly $d^*(\phi) \leq d^{**}(\phi)$. If $d^{**}(\phi) \leq k - 1$, then $d^*(\phi) = d^{**}(\phi)$. The following proposition characterizes the optimal network degree.

Proposition 1 (Network Formation). *Let $k(\phi) \in [0, d^*(\phi)]$ be given by $k \equiv k(\phi) \pmod{d^*(\phi) + 1}$. If $T(d^*(\phi)|\phi) > F(D + 2W)$; then there exists a unique*

⁶This network formation solution concept can be micro-founded by a proposal game. Each bank pays $c > 0$ to make a proposal to another bank. Mutual proposals turn into links, and cost c is refunded to both. One-sided proposals do not turn into links, and c is lost. After being formed, links represent secondary market trades that provide liquidity to banks. The strong Nash equilibria of this game corresponds to strongly stable networks. See Erol and Vohra (2016) for more details.

strongly stable network that consists of $(k - k(\phi))/(d^*(\phi) + 1)$ disjoint cliques of order $d^*(\phi) + 1$ and some more disjoint cliques of smaller orders.⁷ If $T(d^*(\phi)|\phi) < F(D + 2W)$; then the network described above is strongly stable. In any other strongly stable network, all but at most $d^*(\phi)$ banks have degree $d^*(\phi)$.

In the cooperating equilibrium, if less than $S(d^*(\phi)|\phi)$ banks in a clique experience good shocks, then all $d^*(\phi) + 1$ banks in the clique play N . If more than or equal to $S(d^*(\phi)|\phi)$ banks in a clique get good shocks, banks that experience bad shocks play N and banks that experience good shocks play C .

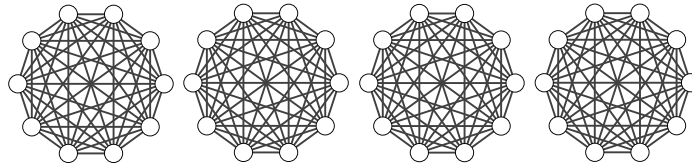


Figure 3: Cliques

Proof.

Here we show that the payoff functions Π can be modified appropriately to satisfy the assumptions that guarantee the main theorem in Erol (2016).

Define $\Pi^*(a_i, f_i, d_i, \theta_i)$ as

$$\Pi^*(a_i, f_i, d_i, \theta_i) = \begin{cases} \Pi(a_i, d_i - f_i, d_i) & \theta_i = G \\ \Pi(N, d_i - f_i, d_i) & \theta_i = B, a_i = N \\ \Pi(N, d_i - f_i, d_i) - 1 & \theta_i = B, a_i = C \end{cases}$$

For this altered function Π^* , even if bad banks were allowed to play C , they would never do so because it is strictly dominant to play N if $\theta_i = B$. Hence, the cooperating equilibrium of the game with payoff function Π^* where bad banks are also allowed to play C corresponds to the cooperating equilibrium of the game with payoff function Π where bad banks are forced to play N .

The assumptions in Erol (2016) are satisfied by Π^* . First, counterparty failures hurt: $\Pi^*(C, f_i, d_i, G)$ must be strictly decreasing in f_i . Notice that $P > D + W + kW$ and $D + W > 0$ ensure that $F(D + W + g_iW)$ stays in the interior of the support of F . Hence $\Pi(C, g_i, d_i)$ is strictly increasing in g_i , which makes $\Pi^*(C, f_i, d_i, G)$ strictly decreasing in f_i . This assumption is relevant for uniqueness, not for existence. Second, N -payoffs are independent of f_i : Given that $\Pi(N, g_i, d_i) = -\kappa_l d_i$ is independent of g_i , this assumption is clearly satisfied. Third, N is a strictly dominant strategy if $\theta_i = B$: This is guaranteed by construction.

⁷See Erol (2016) for the exact order(s) of the remaining smaller clique(s). Describing the remainder of the network requires much heavier notation which we skip for simplicity of the exposition.

Then the main theorem in Erol (2016) can be applied, and the unique strongly stable network is given by disjoint cliques of order $d^*(\phi) + 1$. ■

Figure 3 illustrates the structure of the equilibrium network (in this example, four cliques of order ten—that is each clique has ten banks). Notice that this structure eliminates second order counterparty risk, which is defined as the risk that a bank can incur losses because some of its counterparties choose not to continue because some of their own counterparties do not continue. By forming cliques of an optimal size, banks reach a satiation point in terms of the desired number of counterparties while they do not face any risk of contagion further and beyond what is inevitable, i.e. the risk that their immediate counterparties experience bad shocks. The particular clique structure might seem unrealistic as a description of interbank networks. But as noted at the end of the next section, when there is heterogeneity across banks, they form core-periphery networks and yet the qualitative fragility result that we obtain (see below) from this simple symmetric version remains the same.

If $T(d^*(\phi)|\phi) < F(D + W)$ the available funds from the bank that faces the shock are enough to cover the refinancing need. Then the bank continues even if none of its counterparties continue. As a result, there is no contagion, so there is no second order counterparty risk. If $F(D + W) < T(d^*(\phi)|\phi) < F(D + 2W)$ it is enough that one counterparty continues for the bank in need of refinancing continues. Then the bank continues unless none of its counterparties continue, meaning that there is minimal contagion and there is no second order counterparty risk either. The more interesting case is $T(d^*(\phi)|\phi) > F(D + 2W)$, in which the bank with refinancing needs continues if and only if some certain non-trivial fraction of its counterparties continue. Proposition 1 shows that in this last and more relevant case the strongly stable network is unique.

4. Network Reactions to Banking Regulations

In this section we discuss how the banking network changes in response to changes in the liquidity requirement ϕ , and we show that above a certain critical threshold the network drastically collapses.

In what follows, we ignore integer problems between $d^*(\phi) + 1$ and k and focus on networks that consist of disjoint cliques of order $d^*(\phi) + 1$.⁸

We focus on the expected number of banks that choose N as our main notion of systemic risk. For arbitrary networks this is hard to pin down analytically in closed form. Using Proposition 1 we can compute even the distribution of the number of banks that play N in the unique strongly stable network that is formed.

⁸By assuming that ϕ is selected from a finite subset X of $[0, 1]$, and k is divisible by $\prod_{\phi \in X} (d^*(\phi) + 1)$, integer problems can be eliminated formally. Intuitively, however, for any ϕ the remainder cliques represent a very small fraction of the economy, and they are isolated from the rest of the network, hence they have negligible impact on the comparative statics.

Corollary 1 (Systemic Risk). *For a given ϕ , the expected fraction of banks that play N is*

$$1 - \alpha + \alpha G[S(d^*(\phi)|\phi) - 1, d^*(\phi), \alpha],$$

and the probability that all banks play N is

$$G[S(d^*(\phi)|\phi), d^*(\phi) + 1, \alpha]^{\frac{k}{d^*(\phi)+1}}.$$

Recall that $G[S(d^*(\phi)|\phi) - 1, d^*(\phi), \alpha]$ measures the probability that, given the d^* counterparties that a good bank has, not enough succeed for the bank to continue (bearing in mind that the bank needs $S(d^*(\phi)|\phi)$ of the d^* counterparties to succeed if it is to continue).

If tighter regulations decrease the equilibrium number of counterparties in the network, $d^*(\phi)$, then the level of systemic risk in the economy may increase. In what follows we show that an increase in ϕ reduces d^* smoothly until it reaches a threshold, beyond which d^* suddenly collapses to 0 – a situation that we call an *empty network*.

The optimal number of counterparties for a bank will be given by the degree that maximizes its value function. Define the *resilience* of a bank as

$$R(d|\phi) := d - S(d|\phi) - 1.$$

which increases with the number of counterparts a bank has and decreases with the fragility of the bank (the number of successful counterparties the bank requires to continue). After some algebra, the value $V(d|\phi)$ is given by:

$$V(d|\phi) = A L(\phi) G(R(d|\phi), d, 1 - \alpha) + d \times \left[-\kappa_l - \alpha\kappa_l' G(R(d|\phi), d, 1 - \alpha) + B L(\phi) G(R(d|\phi), d - 1, 1 - \alpha) \right],$$

where

$$A = \alpha\beta(F(D + W) - T_0) = \alpha \left[-\kappa_c + (1 - \eta)(R_2 - 1) + \eta \frac{(D + W)}{P} (mR_2 - 1) \right],$$

$$B = \frac{\alpha^2\beta W}{P}.$$

This characterization of the value function as a function of the network degree, which, according to Proposition 1, is based on cliques, defines the individual optimal number of counterparties that a bank would choose to borrow from in the interbank lending market should it be to successfully face a refinancing need. We describe next the shape of this value function, its maximum $d^*(\phi)$ and how it changes as the regulation parameter ϕ changes.

While the coefficient A governs the behavior of an isolated bank (without any counterparty, or degree $d = 0$), the coefficient that multiplies d nicely decomposes the costs and benefits of having additional counterparties. This expression for the value function is suggestive of discontinuous dynamics. Imagine

first that the probability that a good bank can continue because not enough counterparties are successful, G , is fixed. It is clear that as regulation tightens (ϕ increases) the bank invests less ($L(\phi)$ decreases) and it has less incentives to have a counterparty that helps to refinance a small project in distress. When ϕ is relatively small the coefficient that multiplies d in the previous expression may be positive, and banks want to have all other banks as counterparties (a *complete* network). When ϕ is relatively large the coefficient may be negative, in which case the bank does not want any counterparty. Given the monotonicity of the coefficient that multiplies d when G is fixed, this captures a trivial bang-bang solution. Our model, however, captures a more intricate fragility. We show that the expectation of idiosyncratic bad shocks and the resulting expectation of inefficient cascades of ex-post failures induce banks ex-ante to strategically and jointly reduce the number of their counterparties, until at a certain point the whole network suddenly collapses.

The probability that the bank can obtain enough refinancing funds from counterparties to justify continuation, G , depends, however, on both ϕ and d . How does the value function change with the number of counterparties? This is illustrated in each panel of Figure 4.⁹ Value functions decrease in d when the probability of not obtaining enough funds from counterparties, G , is fixed, and this is so because the bank has to pay κ_l for each additional counterparty. This is clearly the case when d is very small (there are so few counterparties that $G \approx 0$) or when d is very large (there are so many counterparties that $G \approx 1$). When d is intermediate, however, an additional counterparty increases G , and this makes it more likely for a bank to find enough funds to cover refinancing needs. When the increase in the probability of taking the project to maturity is higher than the cost of creating the link, then the value function can increase in this intermediate region. Since G is probabilistically more reactive to ϕ , in this region extra links rapidly become more beneficial up to a probabilistic satiation point. Therefore, V is hump-shaped in this middle region and the peak of the hump is the optimal degree d^* . The peak is achieved at a value of d with large resilience R and G very close to 1.

How does the optimal number of counterparties, d^* , change with regulation, ϕ ? To illustrate this relation we compare the panels in Figure 4. As ϕ increases, the value function for all d and the optimal degree d^* decrease, which generates an initial smooth change in the level of connectivity d^* . This smooth transition occurs until a point in which the maximum of the value function becomes negative. At this tipping point the solution becomes $d^* = 0$, as shown in the last panel of Figure 4.

This analysis shows that a simpler model that features two banks and a bang-bang solution misses important elements that only arise in the presence of networks. Two banks have only one link, and so we cannot study them to determine how banks take the resilience and connectivity of other banks in the

⁹The parameters used to construct Figure 4 are $k = 100$, $\alpha = 0.9$, $\bar{L} = 3000$, $\eta = 1/k$, $W = 10$, $R_1 = 0$, $R_2 = 20$, $m = 1$, $\kappa_d = 1$, $D = 15$, $P = 615$, $\kappa_c = 18.905$, $\kappa_l = 0.03$, $\kappa'_l = 0.02$.

system into account when they choose their own level of connectivity. Changes in regulation modify both the individual benefits of having counterparties and the connectivity choices of other banks and, thus, their likelihood of being successful counterparties. Intuitively, the phase transition happens because as regulation increases and the size of investments declines, each bank is less likely to form counterparties. This implies that each counterparty is less likely to continue or help refinance in the event of a shock. This makes a counterparty less attractive, further reducing the incentive to form links. This introduces a non-linear feedback effect that, after a critical ϕ level is reached, leads to a sudden collapse. Formally, we prove the phase transition in an Online Appendix for a special case.

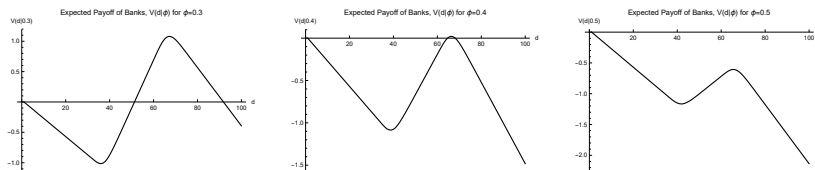


Figure 4: $V(d|\phi)$ as ϕ increases

Naturally, if $k < d^*$, the upper bound $k - 1$ on the number of counterparties binds before reaching the satiation point, and the realized network will be complete (a single clique with all banks as members). After ϕ goes above a certain threshold, the network will suddenly collapse to an empty network. Once the network is empty, no bank continues. Good banks liquidate their projects because the likelihood of a refinancing shock is large compared to the chances of refinancing, and so it does not compensate to pay continuation costs.¹⁰

Remark on heterogeneity across banks: The symmetric clique structure that we propose might seem unrealistic as a description of interbank networks. This particular structure, however, is a simplification of a broader and more general structure that will arise when banks are heterogenous. The main fragility insight we obtain is not an outcome of this simplification. For example if some banks are large in terms of their deposit base while others are small, the network will display a core-periphery structure wherein banks in the periphery are lumped into cliques modulo their links with the core. These cliques will still have a certain size and similar qualitative results would hold in the size of the cliques in the periphery. Another way of introducing heterogeneity is along the

¹⁰Why do banks invest knowing that they will not continue? Banks are indifferent between, on the one hand, investing $L = 0$ and obtaining 0 profits and, on the other hand, investing any other amount and obtaining 0 after liquidation. This is artificial; if there is a very small chance that at the end of the investment stage the project becomes guaranteed to not suffer any refinancing shock at the refinancing stage, then the bank would strictly prefer to invest although it is almost certain that the project will not become guaranteed, and the bank will not continue in the continuation stage.

lines of Farboodi (2015) who assumes that some banks have access to investment opportunities and some do not. In a version of the model in which banks can make ex-ante side payments or links are directed, banks that have investment opportunities would pay for links with banks that don't have the investment opportunities. Moreover, banks with investment opportunities will form cliques among each other, leading to a collection of core-periphery sub-networks. Similar qualitative results will hold in the size of cliques that arise across the banks that have investment opportunities.

5. Optimality of a Network-Conscious Regulator

The analysis so far has been positive and focused on understanding how the policy affects the network structure and the resulting systemic risk. Here we obtain the optimal liquidity requirement that a network-conscious regulator, who understands the effects of regulation in the network structure, would introduce. To simplify the exposition we introduce two assumptions that we relax at the end of this section. First, we assume that the number of banks is not too large. As a consequence, the upper bound $k-1$ on d^* binds before the tipping point, so that when a network exists, it is complete, and then falls to the empty network at the tipping point. Second, we assume that individual deposits of banks are not too large and, thus, they need each other. In particular they are not likely to refinance a project in isolation, so that isolated banks play N (in case of not having any counterparty).

Figure 5 shows that the network collapses from a complete network ($d^* = 100$) to an empty network ($d^* = 0$) at $\phi = 0.25$.¹¹ The second panel in the figure shows the evolution of fragility in the network. As ϕ increases towards 0.25, the number of counterparties that are required for a bank to be successful increases from 50 to 65, at which point the network collapses. The last panel shows the expected fraction of banks that choose not to continue, jumping from 10% (the fraction of bad banks) when the network is complete, to 100% when the network is empty.

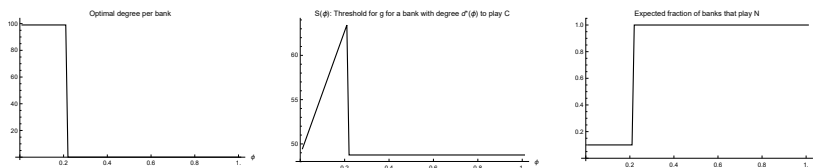


Figure 5: Optimal degree $d^*(\phi)$, induced fragility S , and resulting systemic risk

¹¹The parameters used to construct the Figures 5, 6, 9 are $k = 100$, $\alpha = 0.9$, $\bar{L} = 3000$, $\eta = 1/k$, $W = 10$, $R_1 = 0$, $R_2 = 20$, $m = 1$, $\kappa_d = 1$, $D = 15$, $P = 1025$, $\kappa_c = 18.905$, $\kappa_l = 0.03$, $\kappa'_l = 0.02$.

5.1. Notion of Welfare

Banks have all the bargaining power (so they capture all profits when continuing) and limited liability (so they do not suffer any loss when not continuing). The government has to resort on distortionary bailouts to cover the promises banks made to depositors. As deposits are always repaid, either by the bank or the government, households do not count for welfare.

We can decompose welfare into two parts. One is the *banking component*: the part of welfare that is internalized by banks, which is the surplus from production in case of continuation. The other is the *bailout component*: the part of welfare that is not internalized by banks and that consists of direct and indirect costs the government incurs to cover the depositors in case of no continuation. More specifically,

Banking component: The profits from all bad projects and from all good projects that do not continue are 0. The expected profits from all good projects that continue are $(R_2 - 1)L$. Let g^* denote the total number of good banks that choose to continue: $g^* = |\{n_i : a_i = C, \theta_i = G\}|$. The expected total bank profits in the system is then $(g^* - \mathcal{I})(R_2 - 1)L$, where \mathcal{I} is an indicator function that takes the value 1 if a bank has a refinancing need that cannot be covered (and the project is then lost), and 0 otherwise. We include the total utility costs of forming the network ($k\kappa_l d^*$), the utility costs of managing the network for all good banks that continue ($g^* \kappa'_l d^*$) and the utility costs of managing the continuing projects ($g^* \kappa_c L$). The banking component is then

$$(g^* - \mathcal{I})(R_2 - 1)L - k\kappa_l d^* - g^*(\kappa_c L + \kappa'_l d^*)$$

Bailout component: Let f^* denote the total number of good banks that choose not to continue: $f^* = |\{n_i : a_i = N, \theta_i = G\}|$. This means $k - f^* - g^*$ banks receive bad shocks, and so the government covers $(k - f^* - g^*)L$ due to bad shocks. $f^*(1 - R_1)L$ is the total amount covered by the government due to good banks that decide not to continue. Moreover, $\mathcal{I}L$ is covered due to the refinancing shock. The direct effect of bailouts, not internalized by banks, is the amount of deposits to be covered. The indirect effect comes from distortionary costs to cover these deposits, $\kappa_d \geq 1$ per unit of bailout. The bailout component is then

$$-\kappa_d [(k - f^* - g^*)L + f^*(1 - R_1)L + \mathcal{I}L].$$

Both components are influenced by policy ϕ through three main channels. First is the direct and well-known *investment channel*, as $L(\phi) = \min\{\frac{D}{\phi}, \bar{L}\}$. The project scale has a direct effect both on bank profits and on government bailouts. Second is the *contagion channel*, which is captured by the number of good banks that choose not to continue, f^* , and the good bank that continues but suffers a refinancing need that cannot be covered in the network, \mathcal{I} . Third is the *management channel* which lumps the utility costs of management and link formation incurred by banks.

The number of banks that experience good shocks is $f^* + g^*$. This number is αk in expectation at the regulation stage. We define $f^{**} = \mathbb{E}[f^*]$ and $\mathcal{I}^* = \mathbb{E}[\mathcal{I}]$ wherein expectations are taken at the regulation stage. Expected welfare can

Welfare	Banking component	Bailout component
(Net) Investment channel	$\alpha k(R_2 - 1)L$	$-\kappa_d(1 - \alpha)kL$
Contagion channel	$-f^{**}(R_2 - 1)L - \mathcal{I}^*(mR_2 - 1)L$	$-\kappa_d[f^{**}(1 - R_1)L + \mathcal{I}^*]L$
Management channel	$-k\kappa_l d^* - (\alpha k - f^{**})(\kappa_c L + \kappa_l' d^*)$	—

Table 1: Components of ex-ante welfare

be decomposed into components and channels as follows as in Table 1. Adding up all terms in Table 1 yields ex-ante welfare.

5.2. Optimal regulation and phase transition

Once we compute welfare for different levels of liquidity requirements ϕ , we can identify the level of ϕ^* that maximizes welfare. Figure 6 illustrates how welfare changes in response to ϕ . The optimal liquidity requirement is $\phi^* = 0.25$, which is the threshold point at which the network transitions from complete to empty, as in Figure 5.

Welfare increases in ϕ except at the transition point when the network collapses. More regulation reduces the size of bailouts, except at ϕ^* , at which point the network collapses and there is a sudden jump in the number of bailouts and a sudden decrease in welfare. This result suggests that, if the government has some uncertainty about fundamentals, it might be too risky to try to set the exact optimal policy at the kink since a slight overshooting can result in a large unintended welfare cost through the collapse of interbank lending.

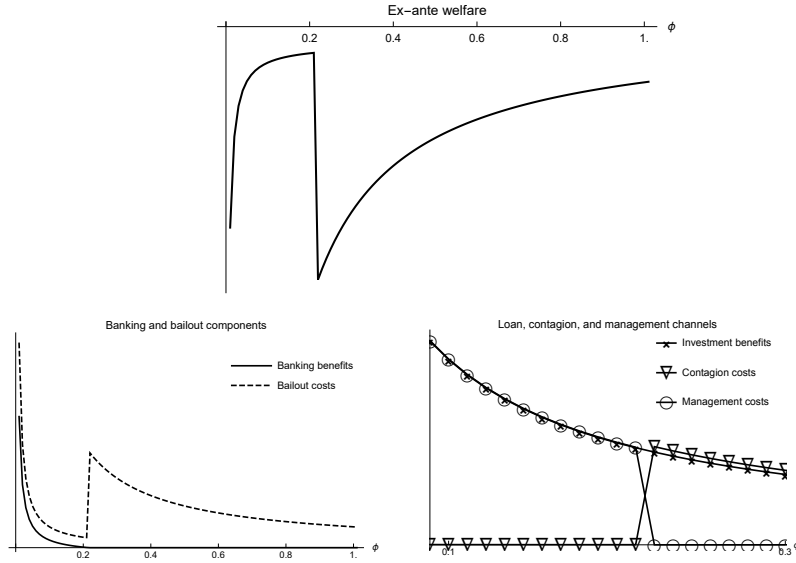


Figure 6: Welfare in ϕ , and its components and channels

Figure 6 also dissects welfare into its components and channels. The banking benefits (the banking component) decline with ϕ because investments decline with ϕ . Since the scale of projects declines, the amount that bailouts have to cover (bailout costs, or the negative of the bailout component) also decline, but only initially, when the network is operating fully. When the network collapses, bailout needs increase discontinuously because no bank chooses to make cross-insurance and all choose to not continue. The government suddenly has to cover smaller projects but for many additional banks. The second panel of the figure also shows the sudden reduction in management costs (the negative of management channel), which are not large enough to compensate the increase in the higher need for bailouts.

5.3. Relaxing Assumptions.

5.3.1. Large number of banks

In the previous welfare analysis we have assumed that k is not too large so that when a network is not empty it is complete. This is so because the largest shock P was sufficiently large to force any bank into default no matter how many counterparties it has. Therefore, there was never a satiation point in terms of the number of desired counterparties. Then banks wanted as many links as possible, hence form a complete network.

Here we show that this assumption is not critical for the results. If k is not as small or P is not as large, there is a satiation point in the number of counterparties needed to satisfy a large probability of surviving a possible refinancing shock (typically a probability close to 1). This generates a dense but not complete network.¹² Changes in regulation now smoothly change the satiation point. Figure 7 shows how the satiation point d^* slowly decreases as regulation becomes tighter. Nonetheless, the network suddenly collapses at the point $\phi = 0.45$.¹³

Notice that in this example welfare starts to fall sharply just before the network's transition point, not exactly at the transition point itself. This is so because in this example the source of the phase transition is the *probabilistic source*. A larger ϕ reduces the size of investments, which decreases fragility S , just as the bank needs a smaller number of successful counterparties to refinance a project in distress. Banks react to this change by decreasing their degree slowly. As ϕ keeps growing, however, it becomes impossible at a certain point to face a refinancing shock with high probability. At this point, the contagion becomes a serious ex-ante problem because a rapidly increasing number of banks are expected to liquidate their good projects. This is seen in the plot of the expected fraction of banks that play N in Figure 7. When ϕ reaches the tipping

¹²Allowing a small P or large k (more specifically $P < D + W + kW$) creates a small technical complication for uniqueness of the strongly stable network. The described network is still strongly stable but it may not be the unique strongly stable network.

¹³The parameters used to construct the Figure 7 are $k = 100$, $\alpha = 0.9$, $\bar{L} = 3000$, $\eta = 1/k$, $W = 10$, $R_1 = 0$, $R_2 = 20$, $m = 1$, $\kappa_d = 1$, $D = 15$, $P = 615$, $\kappa_c = 18.905$, $\kappa_l = 0$, $\kappa'_l = 0.05$.

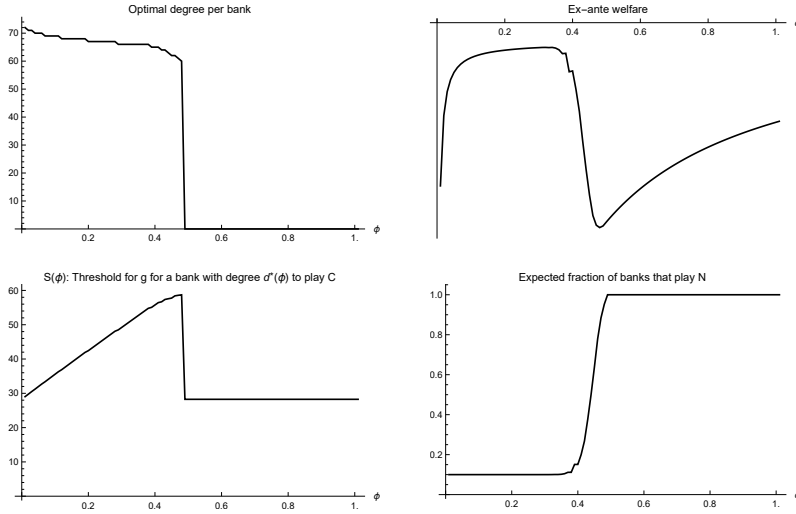


Figure 7: Smooth change in the network structure when the largest shock is not too large, and resulting welfare, fragility, and systemic risk

point, the fragility is insurmountable and there is almost certainty of liquidating a good project with the number of counterparties that banks can afford to have. Banks stop forming links. The optimal degree falls to zero at $\phi = 0.45$ but the payoff to banks and welfare start falling very sharply in advance because of contagion.

5.3.2. Deposits are large enough

When deposits are small, and thus the resources of individual banks are small compared to P , isolated banks are destined to play N , which amplifies bailout costs. Here we show that even when P is relatively small (more specifically, when $D + W > PT_0$) and banks continue even without counterparties, welfare can still feature sharp declines around a transition point at which the networks stop operating. As illustrated in Figure 8, however, the optimal level of regulation in this case occurs not at the transition point of $\phi = 0.5$ (as in the previous cases) but instead is very close to $\phi = 0$.¹⁴

6. Mistakes of a Network-Blind Regulator

Here we discuss the mistakes made by a network-blind regulator, who, when deciding regulation, does not consider that the network structure reacts to ϕ

¹⁴The parameters used to construct the Figure 8 are $k = 100$, $\alpha = 0.9$, $\bar{L} = 3000$, $\eta = 1/k$, $W = 10$, $R_1 = 0$, $R_2 = 20$, $m = 1$, $\kappa_d = 1$, $D = 1$, $P = 1010$, $\kappa_c = 18.791$, $\kappa_l = 0.03$, $\kappa'_l = 0.02$.

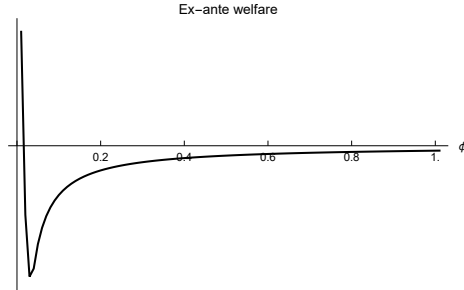


Figure 8: Induced welfare when isolated banks are sufficiently liquid

and naively maximizes welfare by concluding that the observed level of network structure is a given.

More specifically, we have shown that for any arbitrary network in cliques with degrees d and an arbitrary level of liquidity requirement ϕ , there is a welfare level $W(d, \phi)$. Suppose that the economy is at a status quo level of liquidity requirements ϕ_0 (not necessarily optimal at this point) that induce an optimal reaction of banks into a strongly stable network with the optimal degree $d_0 = d^*(\phi_0)$. The status quo welfare is then $W(d^*(\phi_0), \phi_0)$.

Assume now the government decides to adjust ϕ to an optimal level given the observed network degree $d^*(\phi_0)$. If, as has been done in previous analyses, the government takes network reactions into account, then the government maximizes $W(d^*(\phi), \phi)$ over ϕ and ϕ is adjusted to $\phi^{**} = \operatorname{argmax}_{\phi} \{W(d^*(\phi), \phi)\}$ and all banks have degree $d^{**} = d^*(\phi^{**})$.

If, however, the government does not acknowledge how the interbank network reacts to changes in policy, and instead it regards the network that has degrees d_0 as given and fixed, what would the unintended consequences be of such a *network-blind* policy? Formally, the government takes the network in clique structure that has degrees d_0 as given, and chooses a policy $\phi^*(d_0) = \operatorname{argmax}_{\phi} \{W(d_0, \phi)\}$ that maximizes welfare for this fixed network that has degrees d_0 . The network reacts to this policy change and the degrees become $d^*(\phi^*(d_0))$. The realized welfare is then $W(d^*(\phi^*(d_0)), \phi^*(d_0))$. The immediate cost of the network-blind policy is captured by the wedge

$$W(d_0, \phi_0) - W(d^*(\phi^*(d_0)), \phi^*(d_0)),$$

while the opportunity cost of the network-blind policy is captured by the wedge

$$W(d^{**}, \phi^{**}) - W(d^*(\phi^*(d_0)), \phi^*(d_0)).$$

Figure 9, which is based on the benchmark simulation described in the previous section, illustrates the main result. A network-blind regulator who starts at a high status quo ϕ_0 takes the empty network $d^* = 0$ as given. Then, he naively thinks that welfare, in response to regulation, follows the dashed line (labeled by “Network-Blind: Sparse”). In contrast, a network-blind regulator who starts

at a low status quo ϕ_0 takes the complete network $d^* = k - 1$ as given. Then he naively thinks that welfare, in response to regulation, follows the dotted line (labeled by “Network-Blind: Dense”). A network-conscious regulator consider the same solid line as in Figure 6.

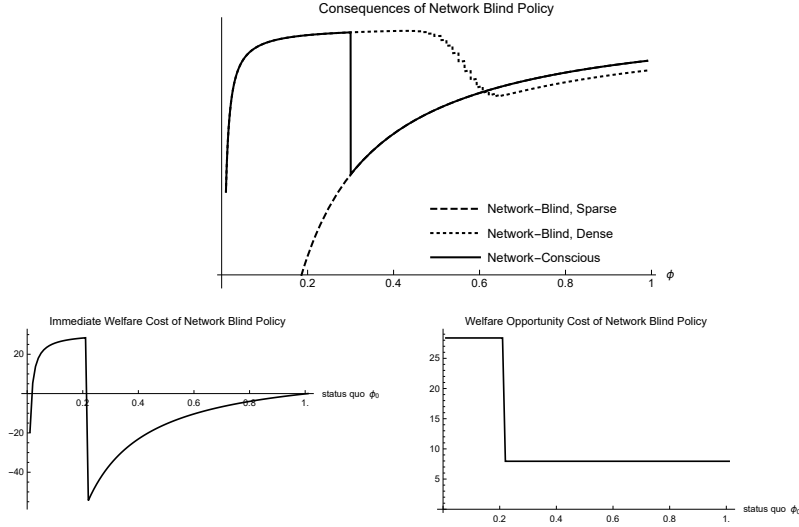


Figure 9: Network-conscious welfare and network-blind welfare for given networks, and the costs network-blind regulation

Imagine that the status quo is given by $\phi_0 = 0.1$, in which the observed network is dense. A network-blind government would think that welfare is maximized at around $\phi = 0.5$. However, once a liquidity requirement of $\phi = 0.5$ is implemented the network reacts by becoming sparse and the realized welfare becomes much lower than both the intended level and the starting status-quo level.

Imagine, in contrast, that the status quo is given by $\phi_0 = 0.8$, in which the observed network is sparse. A network-blind government would think that welfare can be maximized at $\phi = 1$. In this case the welfare improves upon the status quo because the network does not react to this policy. The opportunity cost, however, is still positive since the optimal for a network-conscious policy is a much lower requirement, $\phi^* = 0.25$.

Figure 9 plots the immediate welfare costs and welfare opportunity costs of a network-blind policy for any status-quo ϕ_0 . The first panel shows the comparison of the status-quo welfare level with the (unintended) realization of welfare level under the network-blind optimal policy. For low liquidity requirements the status-quo always shows higher welfare compared to a network-blind optimal policy because the latter implies a network collapse. For high liquidity requirements the status-quo always displays lower welfare compared to a network-blind optimal policy because the status-quo network is already given by the empty

network so that there is no unintended network collapse. The second panel compares the welfare that would be achievable by a network-conscious policy and a network-blind policy for any level of the status-quo. Not surprisingly, this difference is always positive, and it is larger among low levels of status-quo requirements.

7. Conclusions

Discussion about the optimal level of liquidity requirements, and more generally about optimal banking regulations, has generated a fruitful recent debate among academics and policymakers alike, particularly in the light of many creative proposals made by Basel III and the Dodd-Frank Act. Given the prominent role that a dense interbank network and complex counterparty relationships have played during the recent crisis, it is surprising there has been little discussion of how the proposed regulations would affect the density and topology of interbank lending relations and the banking network more generally.

We show that tightening liquidity requirements above a critical threshold can induce a sudden collapse in interbank relationships, thus discontinuously decreasing insurance across banks that face liquidity and refinancing shocks. We argue this is an endogenous network reaction that should be taken into account by regulators who propose further tightening of liquidity requirements. A network-blind tightening in regulation can induce a discontinuous increase in systemic risk: even though the bailouts needed per bank are smaller, more banks know they cannot ride refinancing shocks successfully, and thus they choose to liquidate assets excessively, with the result that the government has to bailout banks that would have been covered by other banks had a network existed.

Our main goal is to highlight the potentially large unforeseen effects of banking regulations on network formation and systemic risks. Although we have focused on liquidity requirements, similar results can be obtained when considering capital requirements and other forms of leverage constraints, as long as these restrictions reduce investments to a level that discourages the formation of networks and the related cross-insurance gains they provide.

The model we have introduced captures the intricate strategic considerations that banks and financial institutions face when they choose counterparties. The model's tractability opens doors to the study of other interesting questions, which we leave for future research. For example, in the model we abstract from potentially interesting trade-offs between liquid and illiquid assets. As a consequence banks choose to invest in illiquid assets as much as possible. Having an optimal amount of liquid assets as a fraction of illiquid assets would allow understanding how networks evolve during periods of scarce liquidity or high returns of illiquid assets.

Similarly, introducing aggregate refinancing shocks, or a correlation among banks' assets (coming, for example, from securitization), also would affect how the network structure evolves. We conjecture that just as there is a level of regulation beyond which a network collapses, there also may be a level of securitization that leads to similar fragility results.

Finally, we have assumed that the government bails out depositors in cases of distress, but we leave open the question of what happens when a government instead bails out banks directly. Indeed, Erol (2016) considers capital injections into banks and shows that the anticipation of bailouts relaxes the market discipline during network formation, and so the network structure is completely dissolved into an arbitrarily interconnected network. He calls this change, which results in higher systemic risk, “network hazard.” It remains to be determined how network hazard interacts with regulation.

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